thm_2Einteger__word_2Eoverflow__add (TMEuhHi2mh5wfmn51X2ZVjmcUtKcG1cnpmn)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 6 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 7 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A\_$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (2)$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (3)$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \quad (4)$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(ty\_$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \quad (5)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \quad (6)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \quad (7)$$

Definition 10 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 11 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \quad (8)$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (9)$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (10)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (11)$$

Definition 12 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (12)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (13)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (14)$$

Definition 15 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 16 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 17 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 18 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ $c_2Ebool_2E_2F_5C

Definition 19 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A_27a : \iota.(ap\ ($c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{15}$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{16}$$

Definition 20 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{17}$$

Definition 21 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 22 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 23 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 24 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 25 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 26 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \tag{20}$$

Definition 27 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a:\iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (21)$$

Let $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ \ A\_27a)}) \quad (22)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (23)$$

Definition 28 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (24)$$

Definition 29 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (25)$$

Definition 30 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 31 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 32 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 33 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 34 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a:\iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 35 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A\_27a:\iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (26)$$

Definition 36 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Definition 37 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A\_27a:\iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Definition 38 *We define* c_2Einteger__word_2Ew2i *to be* $\lambda A\_27a:\iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(a$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{27}$$

Definition 39 *We define* c_2Ewords_2Eword__add *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V$

Definition 40 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{28}$$

Definition 41 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Ewords\_2Eadd\_\_with\_\_carry : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Eadd\_\_with\_\_carry\ A\_27a \in ((ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2))^{(ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2))}) \tag{29}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)) \tag{30}$$

Assume the following.

$$True \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1y \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((\neg((ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}\ A\_27a)\ V1y)))) \Leftrightarrow (((p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0x)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V1y))) \wedge (\neg((p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0x)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ V0x)\ V1y)))))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \tag{39}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1y \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V2carry\_\_in \in 2.((ap\ (c\_2Ewords\_2Eadd\_\_with\_\_carry\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2))\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2)\ V1y)\ V2carry\_\_in))) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2)))\ (\lambda V3unsigned\_\_sum \in ty\_2Enum\_2Enum.(ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2)))\ (\lambda V4result \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ 2\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ 2\ ((ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2))^{2}))\ (\lambda V5carry\_\_out \in 2.(\lambda V6overflow \in 2.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2))\ V4result)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ 2)\ V5carry\_\_out)\ V6overflow)))))\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V4result))\ V3unsigned\_\_sum))))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V1y)))\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V4result)))))))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V3unsigned\_\_sum))))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V1y)))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ V2carry\_\_in)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ \mathsf{c\_2Enum\_2E0}))))))) \tag{40}$$

Theorem 1

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1y \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((\neg((ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}\ A\_27a)\ V1y))))\Leftrightarrow(p\ (ap\ (c\_2Epair\_2ESND\ 2\ 2)\ (ap\ (c\_2Epair\_2ESND\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2))\ (ap\ (c\_2Ewords\_2Eadd\_\_with\_\_carry\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2))\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2)\ V1y)\ \mathsf{c\_2Ebool\_2EF})))))))))$$