

thm_2Einteger__word_2Eoverflow__sub
(TMR8ToZoFXy8maGUX1yYYG9ZREvPc1LsGLb)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ P))))$

Definition 4 We define $c_2Ebool_2E_2T$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x)))\ (\lambda V1x \in 2.V1x)$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}\ P))))$

Definition 6 We define $c_2Earithmetic_2EMODEQ$ to be $\lambda V0n \in ty_2Enum_2Enum.\lambda V1m1 \in ty_2Enum_2Enum$.

Definition 7 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 8 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 9 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (4)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (5)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{ty_2Einteger_2Eint}) \quad (6)$$

Definition 10 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E40\ (t$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (7)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (8)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (9)$$

Definition 11 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 12 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (10)$$

Definition 13 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 14 We define $c_2Einteger_2Eint_sub$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Let $ty_2Efcf_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcf_2Efinite_image\ A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (12)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (13)$$

Let $c_2Efcf_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcf_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (14)$$

Definition 15 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 16 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$.

Definition 18 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (16)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (17)$$

Definition 19 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ V0m)$.

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$.

Definition 21 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap\ (ap\ c_2Ebool_2E_2F_5C\ V0P))$.

Definition 22 We define $c_2Efcf_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum})))$.

Let $ty_2Efcf_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcf_2Ecart\ A0\ A1) \quad (18)$$

Let $c_2Efcf_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcf_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcf_2Efinite_image\ A_27b)})^{(ty_2Efcf_2Ecart\ A_27a\ A_27b)}) \quad (19)$$

Definition 23 We define $c_2Efcf_2Efcf_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcf_2Ecart\ A_27a\ A_27b)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{20}$$

Definition 24 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 25 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 26 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT2\ n))$.

Definition 27 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{21}$$

Definition 28 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_2Ebool_2ECOND\ t1\ t2))))$.

Definition 29 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ESBIT\ b)\ n)))$.

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \tag{22}$$

Definition 30 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ (c_2Ewords_2Ew2n\ w)))$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \tag{23}$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \tag{24}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{25}$$

Definition 31 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT1\ n))$.

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{26}$$

Definition 32 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Ebit_2EDIV_2EXP\ x\ n)$.

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (27)$$

Definition 33 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 34 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \lambda V$

Definition 35 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap$

Definition 36 We define c_2EfcP_2EFCP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a^{ty_2Enum_2Enum}). (ap$

Definition 37 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. (ap (c_2EfcP_2EFCP$

Definition 38 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).$

Definition 39 We define $c_2Ewords_2Eword_2msb$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).$

Definition 40 We define $c_2Einteger_2word_2Ew2i$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (28)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (29)$$

Definition 41 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 42 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in$

Definition 43 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 44 We define $c_2Eprim_2rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2E$

Definition 45 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2) \quad (30)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (31)$$

Definition 46 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27b$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (32)$$

Definition 47 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Ewords_2Eadd_with_carry : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Eadd_with_carry \\ A_27a \in ((ty_2Epair_2Eprod\ (ty_2Efc_2Ecart\ 2\ A_27a)\ (ty_2Epair_2Eprod \\ 2\ 2))^{(ty_2Epair_2Eprod\ (ty_2Efc_2Ecart\ 2\ A_27a)\ (ty_2Epair_2Eprod\ (ty_2Efc_2Ecart\ 2\ A_27a)\ 2))}) \end{aligned} \quad (33)$$

Definition 48 We define $c_2Ewords_2Eword_add$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1$

Definition 49 We define $c_2Ewords_2Eword_sub$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (\\ ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \end{aligned} \quad (34)$$

Definition 50 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewords_2E2w\ A_27a)\ (ap\ (c_2Ew$

Definition 51 We define $c_2Ewords_2Eword_1comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 52 We define $c_2Ewords_2Eword_mul$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1$

Definition 53 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ c_2Enum_2E0) = V0m)) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ V1n)\ V0m)))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ \forall V2p \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\\
& (p (ap (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1y)) \Rightarrow (((ap (ap c_2Earithmetic_2EMOD \\
& V0x) V1y) = V0x) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0x) V1y)))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1e0 \in ty_2Enum_2Enum. \\
& (\forall V2e1 \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow ((p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) \\
& V1e0) V2e1)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD V1e0) V0n) = (ap (ap \\
& c_2Earithmetic_2EMOD V2e1) V0n)))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x0 \in ty_2Enum_2Enum. \\
& (\forall V2x1 \in ty_2Enum_2Enum. (\forall V3y0 \in ty_2Enum_2Enum. \\
& (\forall V4y1 \in ty_2Enum_2Enum. ((p (ap (ap (ap c_2Earithmetic_2EMODEQ \\
& V0n) V1x0) V2x1)) \Rightarrow ((p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) V3y0) \\
& V4y1)) \Rightarrow (p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) (ap (ap c_2Earithmetic_2E_2B \\
& V1x0) V3y0)) (ap (ap c_2Earithmetic_2E_2B V2x1) V4y1))))))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (\\
& p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) V1x) V1x))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow (p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) \\
& V0n) c_2Enum_2E0)))) \\
& \hspace{15em} (42)
\end{aligned}$$

Assume the following.

$$\text{True} \hspace{15em} (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \\
& \hspace{15em} (44)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \hspace{15em} (45)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \hspace{15em} (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in A_27a. ((ap\ (ap\ (c_2Ebool_2ELET \\ & A_27a\ A_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (53)$$

Assume the following.

$$((\neg(True \Leftrightarrow False)) \wedge (\neg(False \Leftrightarrow True))) \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (56)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in (ty_2Efc_2Ecart \\ & 2 A_27a). (\forall V1y \in (ty_2Efc_2Ecart 2 A_27a). ((\neg((ap (c_2Einteger_word_2Ew2i \\ & A_27a) (ap (ap (c_2Ewords_2Eword_sub A_27a) V0x) V1y)) = (ap (ap \\ & c_2Einteger_2Eint_sub (ap (c_2Einteger_word_2Ew2i A_27a) \\ & V0x)) (ap (c_2Einteger_word_2Ew2i A_27a) V1y)))) \Leftrightarrow ((\neg((p (ap \\ & (c_2Ewords_2Eword_msb A_27a) V0x)) \Leftrightarrow (p (ap (c_2Ewords_2Eword_msb \\ & A_27a) V1y)))))) \wedge (\neg((p (ap (c_2Ewords_2Eword_msb A_27a) V0x)) \Leftrightarrow \\ & (p (ap (c_2Ewords_2Eword_msb A_27a) (ap (ap (c_2Ewords_2Eword_sub \\ & A_27a) V0x) V1y)))))))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))))) \\
& \tag{63}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))))) \\
& \tag{64}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V0n) \\
& V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n)) (ap c_2Earithmetic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB \\
& c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0))) \\
& \tag{65}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\
& \forall V0x \in A.27a. (\forall V1y \in A.27b. ((p (c_2Epair_2ESND A.27a \\
& A.27b) (ap (ap (c_2Epair_2E_2C A.27a A.27b) V0x) V1y)) = V1y))) \\
& \tag{66}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (79)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in (ty_2EfcP_2Ecart \\ & \quad 2 A.27a).(\forall V1y \in (ty_2EfcP_2Ecart 2 A.27a).(\forall V2carry_in \in \\ & \quad 2.((ap (c.2Ewords_2Eadd_with_carry A.27a) (ap (ap (c.2Epair_2E_2C \\ & \quad (ty_2EfcP_2Ecart 2 A.27a) (ty_2Epair_2Eprod (ty_2EfcP_2Ecart \\ & \quad 2 A.27a) 2)) V0x) (ap (ap (c.2Epair_2E_2C (ty_2EfcP_2Ecart 2 \\ & \quad A.27a) 2) V1y) V2carry_in))) = (ap (ap (c.2Ebool_2ELET ty_2Enum_2Enum \\ & \quad (ty_2Epair_2Eprod (ty_2EfcP_2Ecart 2 A.27a) (ty_2Epair_2Eprod \\ & \quad 2 2))) (\lambda V3unsigned_sum \in ty_2Enum_2Enum.(ap (ap (c.2Ebool_2ELET \\ & \quad (ty_2EfcP_2Ecart 2 A.27a) (ty_2Epair_2Eprod (ty_2EfcP_2Ecart \\ & \quad 2 A.27a) (ty_2Epair_2Eprod 2 2))) (\lambda V4result \in (ty_2EfcP_2Ecart \\ & \quad 2 A.27a).(ap (ap (c.2Ebool_2ELET 2 (ty_2Epair_2Eprod (ty_2EfcP_2Ecart \\ & \quad 2 A.27a) (ty_2Epair_2Eprod 2 2))) (ap (ap (c.2Ebool_2ELET 2 \\ & \quad ((ty_2Epair_2Eprod (ty_2EfcP_2Ecart 2 A.27a) (ty_2Epair_2Eprod \\ & \quad 2 2)))^2)) (\lambda V5carry_out \in 2.(\lambda V6overflow \in 2.(ap (ap \\ & \quad (c.2Epair_2E_2C (ty_2EfcP_2Ecart 2 A.27a) (ty_2Epair_2Eprod \\ & \quad 2 2)) V4result) (ap (ap (c.2Epair_2E_2C 2 2) V5carry_out) V6overflow)))))) \\ & \quad (ap c.2Ebool_2E_7E (ap (ap (c.2Emin_2E_3D ty_2Enum_2Enum) (ap \\ & \quad (c.2Ewords_2Ew2n A.27a) V4result)) V3unsigned_sum)))) (ap (\\ & \quad ap c.2Ebool_2E_5C (ap (ap (c.2Emin_2E_3D 2) (ap (c.2Ewords_2Eword_msb \\ & \quad A.27a) V0x)) (ap (c.2Ewords_2Eword_msb A.27a) V1y)) (ap c.2Ebool_2E_7E \\ & \quad (ap (ap (c.2Emin_2E_3D 2) (ap (c.2Ewords_2Eword_msb A.27a) V0x)) \\ & \quad (ap (c.2Ewords_2Eword_msb A.27a) V4result)))))) (ap (c.2Ewords_2En2w \\ & \quad A.27a) V3unsigned_sum)))) (ap (ap c.2Earithmetic_2E_2B (ap (\\ & \quad ap c.2Earithmetic_2E_2B (ap (c.2Ewords_2Ew2n A.27a) V0x)) (ap \\ & \quad (c.2Ewords_2Ew2n A.27a) V1y)) (ap (ap (ap (c.2Ebool_2ECOND ty_2Enum_2Enum) \\ & \quad V2carry_in) (ap c.2Earithmetic_2ENUMERAL (ap c.2Earithmetic_2EBIT1 \\ & \quad c.2Earithmetic_2EZERO))) c.2Enum_2E0)))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (p (ap (ap c.2Eprim_rec_2E_3C c.2Enum_2E0) \\ & \quad (ap (c.2Ewords_2Edimword A.27a) (c.2Ebool_2Ethe_value A.27a)))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ & \quad (ap (c.2Ewords_2Ew2n A.27a) (ap (c.2Ewords_2En2w A.27a) V0n))) = \\ & \quad (ap (ap c.2Earithmetic_2EMOD V0n) (ap (c.2Ewords_2Edimword A.27a) \\ & \quad (c.2Ebool_2Ethe_value A.27a)))) \end{aligned} \quad (82)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (c_2Ewords_2Ew2n\ A_27a)\ V0w)) = V0w)) \quad (83)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap\ (c_2Ewords_2En2w\ A_27a)\ V0m) = (ap\ (c_2Ewords_2En2w\ A_27a)\ V1n)) \Leftrightarrow ((ap\ (ap\ c_2Earithmetic_2EMOD\ V0m)\ (ap\ (c_2Ewords_2Edimword\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))) = (ap\ (ap\ c_2Earithmetic_2EMOD\ V1n)\ (ap\ (c_2Ewords_2Edimword\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))))))) \quad (84)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Ewords_2Eword_2comp\ A_27a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) = (c_2Ewords_2Eword_T\ A_27a)) \quad (85)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).((p\ (ap\ (c_2Ewords_2Eword_msb\ A_27a)\ (ap\ (c_2Ewords_2Eword_1comp\ A_27a)\ V0w))) \Leftrightarrow (\neg(p\ (ap\ (c_2Ewords_2Eword_msb\ A_27a)\ V0w)))))) \quad (86)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (c_2Ewords_2Ew2n\ A_27a)\ V0a))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Enum_2Enum)\ (ap\ (ap\ (c_2Emin_2E_3D\ (ty_2EfcP_2Ecart\ 2\ A_27a)\ V0a)\ (c_2Ewords_2Eword_T\ A_27a)))\ (ap\ (c_2Ewords_2Edimword\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))\ (ap\ (c_2Ewords_2Ew2n\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ V0a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))))))) \quad (87)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (c_2Ewords_2Eword_1comp\ A_27a)\ (ap\ (c_2Ewords_2Eword_1comp\ A_27a)\ V0a)) = V0a)) \quad (88)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((\forall V0w \in (ty_2EfcP_2Ecart \\ 2\ A.27a).((ap\ (ap\ (c_2Ewords_2Eword_add\ A.27a)\ V0w)\ (ap\ (c_2Ewords_2En2w \\ A.27a)\ c_2Enum_2E0)) = V0w)) \wedge & (\forall V1w \in (ty_2EfcP_2Ecart\ 2 \\ A.27a).((ap\ (ap\ (c_2Ewords_2Eword_add\ A.27a)\ (ap\ (c_2Ewords_2En2w \\ A.27a)\ c_2Enum_2E0))\ V1w) = V1w))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0v \in (ty_2EfcP_2Ecart \\ 2\ A.27a).(\forall V1w \in (ty_2EfcP_2Ecart\ 2\ A.27a).(\forall V2x \in & \\ (ty_2EfcP_2Ecart\ 2\ A.27a).((ap\ (ap\ (c_2Ewords_2Eword_add\ A.27a) \\ V0v)\ (ap\ (ap\ (c_2Ewords_2Eword_add\ A.27a)\ V1w)\ V2x)) = (ap\ (ap\ (\\ c_2Ewords_2Eword_add\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_add \\ A.27a)\ V0v)\ V1w))\ V2x)))))) \end{aligned} \quad (90)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0v \in (ty_2EfcP_2Ecart \\ 2\ A.27a).(\forall V1w \in (ty_2EfcP_2Ecart\ 2\ A.27a).((ap\ (ap\ (c_2Ewords_2Eword_add \\ A.27a)\ V0v)\ V1w) = (ap\ (ap\ (c_2Ewords_2Eword_add\ A.27a)\ V1w)\ V0v)))) \end{aligned} \quad (91)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0v \in (ty_2EfcP_2Ecart \\ 2\ A.27a).(\forall V1w \in (ty_2EfcP_2Ecart\ 2\ A.27a).(((ap\ (ap\ (\\ c_2Ewords_2Eword_mul\ A.27a)\ (ap\ (c_2Ewords_2En2w\ A.27a)\ c_2Enum_2E0)) \\ V0v) = (ap\ (c_2Ewords_2En2w\ A.27a)\ c_2Enum_2E0)) \wedge & (((ap\ (ap\ (c_2Ewords_2Eword_mul \\ A.27a)\ V0v)\ (ap\ (c_2Ewords_2En2w\ A.27a)\ c_2Enum_2E0)) = (ap\ (c_2Ewords_2En2w \\ A.27a)\ c_2Enum_2E0)) \wedge & (((ap\ (ap\ (c_2Ewords_2Eword_mul\ A.27a) \\ (ap\ (c_2Ewords_2En2w\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V0v) = V0v) \wedge & \\ (((ap\ (ap\ (c_2Ewords_2Eword_mul\ A.27a)\ V0v)\ (ap\ (c_2Ewords_2En2w \\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ c_2Earithmetic_2EZERO)))))) = V0v) \wedge & (((ap\ (ap\ (c_2Ewords_2Eword_mul \\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_add\ A.27a)\ V0v)\ (ap\ (c_2Ewords_2En2w \\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ c_2Earithmetic_2EZERO))))))\ V1w) = (ap\ (ap\ (c_2Ewords_2Eword_add \\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A.27a)\ V0v)\ V1w))\ V1w)) \wedge & \\ (((ap\ (ap\ (c_2Ewords_2Eword_mul\ A.27a)\ V0v)\ (ap\ (ap\ (c_2Ewords_2Eword_add \\ A.27a)\ V1w)\ (ap\ (c_2Ewords_2En2w\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) = (ap\ (\\ ap\ (c_2Ewords_2Eword_add\ A.27a)\ V0v)\ (ap\ (ap\ (c_2Ewords_2Eword_mul \\ A.27a)\ V0v)\ V1w))))))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart \\ 2\ A.27a).((ap\ (c_2Ewords_2Eword_1comp\ A.27a)\ V0w) = (ap\ (ap\ (\\ c_2Ewords_2Eword_sub\ A.27a)\ (ap\ (c_2Ewords_2Eword_2comp\ A.27a) \\ V0w))\ (ap\ (c_2Ewords_2En2w\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (93)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c_2Ewords_2Eword_1comp \\ A.27a)\ (c_2Ewords_2Eword_T\ A.27a)) = (ap\ (c_2Ewords_2En2w\ A.27a) \\ c_2Enum_2E0)) \end{aligned} \quad (94)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart \\ 2\ A.27a).((ap\ (c_2Ewords_2Eword_2comp\ A.27a)\ V0w) = (ap\ (ap\ (\\ c_2Ewords_2Eword_mul\ A.27a)\ (ap\ (c_2Ewords_2Eword_2comp\ A.27a) \\ (ap\ (c_2Ewords_2En2w\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V0w))) \end{aligned} \quad (95)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ (ap\ (ap\ (c_2Ewords_2Eword_add\ A.27a)\ (ap\ (c_2Ewords_2Eword_2comp \\ A.27a)\ (ap\ (c_2Ewords_2En2w\ A.27a)\ V0m)))\ (ap\ (c_2Ewords_2Eword_2comp \\ A.27a)\ (ap\ (c_2Ewords_2En2w\ A.27a)\ V1n))) = (ap\ (c_2Ewords_2Eword_2comp \\ A.27a)\ (ap\ (c_2Ewords_2En2w\ A.27a)\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ V0m)\ V1n)))))) \wedge (\forall V2m \in ty_2Enum_2Enum.(\forall V3n \in ty_2Enum_2Enum. \\ ((ap\ (ap\ (c_2Ewords_2Eword_add\ A.27b)\ (ap\ (c_2Ewords_2En2w\ A.27b) \\ V2m))\ (ap\ (c_2Ewords_2Eword_2comp\ A.27b)\ (ap\ (c_2Ewords_2En2w \\ A.27b)\ V3n))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2EfcP_2Ecart\ 2 \\ A.27b))\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V3n)\ V2m))\ (ap\ (c_2Ewords_2En2w \\ A.27b)\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V2m)\ V3n)))\ (ap\ (c_2Ewords_2Eword_2comp \\ A.27b)\ (ap\ (c_2Ewords_2En2w\ A.27b)\ (ap\ (ap\ c_2Earithmetic_2E_2D \\ V3n)\ V2m))))))))) \end{aligned} \quad (96)$$

Theorem 1

$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Efc_2Ecart$
 $2\ A_27a).(\forall V1y \in (ty_2Efc_2Ecart\ 2\ A_27a).((\neg((ap\ (c_2Einteger_word_2Ew2i$
 $A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_sub\ A_27a)\ V0x)\ V1y)) = (ap\ (ap$
 $c_2Einteger_2Eint_sub\ (ap\ (c_2Einteger_word_2Ew2i\ A_27a)$
 $V0x))\ (ap\ (c_2Einteger_word_2Ew2i\ A_27a)\ V1y)))) \Leftrightarrow (p\ (ap\ (c_2Epair_2ESND$
 $2\ 2)\ (ap\ (c_2Epair_2ESND\ (ty_2Efc_2Ecart\ 2\ A_27a)\ (ty_2Epair_2Eprod$
 $2\ 2))\ (ap\ (c_2Ewords_2Eadd_with_carry\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C$
 $(ty_2Efc_2Ecart\ 2\ A_27a)\ (ty_2Epair_2Eprod\ (ty_2Efc_2Ecart$
 $2\ A_27a)\ 2))\ V0x)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Efc_2Ecart\ 2$
 $A_27a)\ 2)\ (ap\ (c_2Ewords_2Eword_1comp\ A_27a)\ V1y))\ c_2Ebool_2ET))))))$