thm_2Einteger__word_2Ew2i__11__lift (TM-GrE1iAgYjTQV2Mv1WgzwdEgDRf7mz9zQd)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{4}$$

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota{\Rightarrow}\iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota{\Rightarrow}\iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}}$

Definition 5 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (\text{c\_2Emin\_2E\_40}\ (ty\_$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{5}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$
(6)

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})})$$
(7)

Definition 6 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 7 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$
(8)

Definition 8 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(9)

Definition 9 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty}$$
(10)

Definition 10 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})$$
(11)

Definition 11 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum})$$
(12)

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0)$$
(13)

2

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \qquad (14)$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (\atop ty\_2Ebool\_2Eitself\ A\_27a) \qquad (15)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \qquad (16)$$

Definition 12 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (17)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (18)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (19)$$

Definition 16 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 18 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 19 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 20 *We define* c_2Efcp_2Efinite_index *to be* $\lambda A\_27a : \iota.(ap\ ($c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart \atop A0\ A1) \qquad (20)$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart \atop A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \qquad (21)$$

3

Definition 21 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (22)$$

Definition 22 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 23 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 24 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 25 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(23)$$

Definition 26 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 27 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ ($c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum})$$
$$(24)$$

Definition 28 *We define* c_2Ewords_2Ew2n *to be* $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).(ap\ (ap\ $

Let $c\_2Ewords\_2Edimword : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)})$$
$$(25)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(26)$$

Definition 29 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(27)$$

Definition 30 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(28)$$

4

Definition 31 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 32 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 33 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 34 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 35 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 36 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Definition 37 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Definition 38 *We define* c_2Einteger__word_2Ew2i *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(a$

Definition 39 *We define* c_2Einteger_2ENum *to be* $\lambda V0i \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 ty_2E

Definition 40 *We define* c_2Einteger__word_2Ei2w *to be* $\lambda A\_27a : \iota.\lambda V0i \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$

Definition 41 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Definition 42 *We define* c_2Ebit_2ESIGN__EXTEND *to be* $\lambda V0l \in ty\_2Enum\_2Enum.\lambda V1h \in ty\_2Enum\_2$

Definition 43 *We define* c_2Ewords_2Esw2sw *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_2$

Let $c\_2Einteger\_\_word\_2EINT\_\_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Einteger\_\_word\_2EINT\_\_MIN \\ A\_27a \in (ty\_2Einteger\_2Eint^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{29}$$

Let $c\_2Einteger\_\_word\_2EINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Einteger\_\_word\_2EINT\_\_MAX \\ A\_27a \in (ty\_2Einteger\_2Eint^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{30}$$

Definition 44 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Definition 45 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 46 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enumeral\_2EiSUB : \iota$ be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^2) \tag{31}$$

Assume the following.

$$True \tag{32}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p \\ V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2))))))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(False{\Rightarrow}(p\ V0t))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t)\wedge True){\Leftrightarrow} \\ (p\ V0t))\wedge(((False\wedge(p\ V0t)){\Leftrightarrow}False)\wedge((((p\ V0t)\wedge False){\Leftrightarrow}False)\wedge \\ (((p\ V0t)\wedge(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t)){\Leftrightarrow}True)\wedge((((p\ V0t)\vee True){\Leftrightarrow}True)\wedge \\ (((False\vee(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t)\vee False){\Leftrightarrow}(p\ V0t))\wedge(((p\ V0t)\vee \\ (p\ V0t)){\Leftrightarrow}(p\ V0t)))))))$$
$$\tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Rightarrow}True){\Leftrightarrow} \\ True)\wedge(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge((\\ (p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t))))))))) \tag{38}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t)))\wedge(((\neg True){\Leftrightarrow}False)\wedge \\ ((\neg False){\Leftrightarrow}True))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(V0x = V0x)) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow} \\ True)) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{42}$$

6

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\quad (43)$$
$$p\ V0t)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.(((ap\ (ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})$$
$$V0t1)\ V1t2) = V1t2))))$$

$$(44)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in$$
$$A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in$$
$$(2^{A\_27a}).((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow$$
$$((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A\_27a.(p\ (\quad (46)$$
$$ap\ V1Q\ V4x)))))))$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee$$
$$(p\ V1B))))) \quad (47)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow$$
$$((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (48)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_lt}\ V0x)\ V1y)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_add}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))$$
$$V1y))))) \quad (49)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(\forall V2z \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$V0x)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_add}\ V1y)\ V2z))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_add}\ V0x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$V2z)))\ V1y)))))) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((p\ (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_le}\ V0x)\ V1y)) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_le}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \textsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$V1y)\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_neg}\ V0x)))))))))$$

$$(51)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Einteger\_2Eint}).((\forall V1n \in ty\_2Enum\_2Enum.$$
$$(p\ (ap\ V0P\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)))) \Leftrightarrow (\forall V2x \in$$
$$ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\textsf{c\_2Enum\_2E0}))\ V2x)) \Rightarrow (p\ (ap\ V0P\ V2x))))))))$$

$$(52)$$

Assume the following.

$$(\forall V0y \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.$$
$$((ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}\ V1x)\ V0y) = (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$V0y)\ V1x))))$$

$$(53)$$

Assume the following.

$$(\forall V0z \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(\forall V2x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$V2x)\ (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V0z)) = (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}\ V2x)\ V1y))\ V0z)))))$$

$$(54)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \textsf{c\_2Enum\_2E0}))\ V0x) = V0x))$$

$$(55)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \textsf{c\_2Enum\_2E0})) = V0x))$$

$$(56)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_mul}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))))\ V0x) = V0x))$$

$$(57)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(\forall V2z \in ty\_2Einteger\_2Eint.((ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_mul}$$
$$(ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_mul}$$
$$V1y)\ V2z))))))$$

$$(58)$$

8

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg} \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_add} \; V0x)$$
$$V1y)) = (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_add} \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$V0x)) \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg} \; V1y)))))$$

(59)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_mul}$$
$$(ap \; c\_2Einteger\_2Eint\_\_of\_\_num \; \mathsf{c\_2Enum\_2E0})) \; V0x) = (ap \; c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})))$$

(60)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg} \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_mul} \; V0x)$$
$$V1y)) = (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_mul} \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$V0x)) \; V1y))))$$

(61)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg} \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_mul} \; V0x)$$
$$V1y)) = (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_mul} \; V0x) \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$V1y)))))$$

(62)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap \; \mathsf{c\_2Einteger\_2Eint\_\_neg} \; V0x)) = V0x))$$

(63)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((\neg(p \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_le} \; V0x) \; V1y))) \Leftrightarrow (p \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_lt}$$
$$V1y) \; V0x)))))$$

(64)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(\forall V2z \in ty\_2Einteger\_2Eint.(((p \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$V0x) \; V1y)) \wedge (p \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_le} \; V1y) \; V2z))) \Rightarrow (p \; (ap$$
$$(ap \; \mathsf{c\_2Einteger\_2Eint\_\_le} \; V0x) \; V2z))))))$$

(65)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((p \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_le} \; V1y) \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_add}$$
$$V0x) \; V1y))) \Leftrightarrow (p \; (ap \; (ap \; \mathsf{c\_2Einteger\_2Eint\_\_le} \; (ap \; c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) \; V0x)))))$$

(66)

9

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(p \ (ap \ (ap \ \textsf{c\_2Einteger\_2Eint\_\_le} \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num \\
V0m)) \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num \ V1n))) \Leftrightarrow (p \ (ap \ (ap \ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \\
V0m) \ V1n)))))$$

$$(67)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\
((p \ (ap \ (ap \ \textsf{c\_2Einteger\_2Eint\_\_le} \ (ap \ \textsf{c\_2Einteger\_2Eint\_\_neg} \\
V0x)) \ (ap \ \textsf{c\_2Einteger\_2Eint\_\_neg} \ V1y))) \Leftrightarrow (p \ (ap \ (ap \ \textsf{c\_2Einteger\_2Eint\_\_le} \\
V1y) \ V0x)))))$$

$$(68)$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1n \in ty\_2Enum\_2Enum.$$
$$(\forall V2m \in ty\_2Enum\_2Enum.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0p) = V0p) \wedge (((}$$
$$ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0p)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) = V0p) \wedge (((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \wedge$$
$$(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0p)) =$$
$$V0p) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m))) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V2m))))) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V2m)))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap$$
$$(ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V2m)\ V1n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n)$$
$$V2m))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2m)$$
$$V1n)))))) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1n))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V2m))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2m)$$
$$V1n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n)$$
$$V2m)))))) \wedge ((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m)))) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2m)))))))))))))))))))$$

$$(69)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow$$
$$True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge(((p$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge(((p$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap$$
$$c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap$$
$$\mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))) \ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap$$
$$\mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))) \ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap$$
$$c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ ($$
$$ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))))) \Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))))) \Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1m))))\Leftrightarrow True)\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))))))))))))))))))$$

$$(70)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0i \in ty\_2Einteger\_2Eint.$$
$$((((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ (c\_2Einteger\_\_word\_2EINT\_\_MIN$$
$$A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ V0i)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$V0i)\ (ap\ (c\_2Einteger\_\_word\_2EINT\_\_MAX\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27a))))) \Rightarrow ((ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}\ A\_27a)\ (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ei2w}$$
$$A\_27a)\ V0i)) = V0i)))$$

$$(71)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart$$
$$2\ A\_27a).(\exists V1i \in ty\_2Einteger\_2Eint.((V0w = (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ei2w}$$
$$A\_27a)\ V1i)) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ (c\_2Einteger\_\_word\_2EINT\_\_MIN$$
$$A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ V1i)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$V1i)\ (ap\ (c\_2Einteger\_\_word\_2EINT\_\_MAX\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27a))))))))$$

$$(72)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0j \in ty\_2Einteger\_2Eint.(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$(ap\ (c\_2Einteger\_\_word\_2EINT\_\_MIN\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27b)))\ V0j)) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0j)\ (ap\ (c\_2Einteger\_\_word\_2EINT\_\_MAX$$
$$A\_27b)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (c\_2Efcp\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27b)))$$
$$(ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))))) \Rightarrow$$
$$((ap\ (\mathsf{c\_2Ewords\_2Esw2sw}\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ei2w}$$
$$A\_27b)\ V0j)) = (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ei2w}\ A\_27a)\ V0j))))$$

$$(73)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)$$
$$(c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)$$
$$(c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$(ap\ (c\_2Einteger\_\_word\_2EINT\_\_MIN\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27b)))\ (ap\ (c\_2Einteger\_\_word\_2EINT\_\_MIN\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27a)))))$$

$$(74)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)$$
$$(c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)$$
$$(c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$(ap\ (c\_2Einteger\_\_word\_2EINT\_\_MAX\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27a)))\ (ap\ (c\_2Einteger\_\_word\_2EINT\_\_MAX\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27b)))))$$

(75)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow$$
$$(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))))$$

(76)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n)) \Leftrightarrow$$
$$True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge$$
$$((((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))))))))))$$

(77)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)$$
$$V1m)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V1m)\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ (ap\ c\_2Enumeral\_2EiSUB$$
$$\mathsf{c\_2Ebool\_2ET})\ V0n)\ V1m)))\ \mathsf{c\_2Enum\_2E0}))))$$

$$(78)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{79}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow$$
$$(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{81}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow$$
$$((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{82}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{83}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg($$
$$p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee$$
$$((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))))$$

$$(84)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee$$
$$(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))))$$

$$(85)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge$$
$$((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))$$

$$(86)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \quad (87)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \quad (88)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \quad (89)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \quad (90)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \quad (91)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \quad (92)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \quad (93)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow (\forall V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1b \in$$
$$(ty\_2Efcp\_2Ecart\ 2\ A\_27b).(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))$$
$$(ap\ (c\_2Efcp\_2Edimindex\ A\_27c)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27c))))\wedge$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)$$
$$(c\_2Ebool\_2Ethe\_\_value\ A\_27b)))\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27c)$$
$$(c\_2Ebool\_2Ethe\_\_value\ A\_27c)))))\Rightarrow(((ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}$$
$$A\_27a)\ V0a) = (ap\ (\mathsf{c\_2Einteger\_\_word\_2Ew2i}\ A\_27b)\ V1b))\Leftrightarrow((ap\ ($$
$$\mathsf{c\_2Ewords\_2Esw2sw}\ A\_27a\ A\_27c)\ V0a) = (ap\ (\mathsf{c\_2Ewords\_2Esw2sw}\ A\_27b$$
$$A\_27c)\ V1b))))))$$