

thm_2Integer__word_2Ew2i__INT__MINw
 (TMQtTsR-
 CQn6opBu1d3T1bWMAEAv37sWynjn)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow q Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty (ty_2EfcP_2Efinite_image\ A0) \tag{4}$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty (ty_2Ebool_2Eitself\ A0) \tag{5}$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (6)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcp_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (7)$$

Definition 8 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (9)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($

Definition 10 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ ($

Definition 12 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E2F_5C\ ($

Definition 14 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E40\ (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \quad (10)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \quad (11)$$

Definition 15 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Definition 16 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 17 We define `c_2Earithmetic_2EBIT2` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 18 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `c_2Earithmetic_2EEXP` : ι be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 19 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 20 We define `c_2Ebit_2ESBIT` to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebo$

Let `c_2Esum_num_2ESUM` : ι be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 21 We define `c_2Ewords_2Ew2n` to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap (ap c$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (15)$$

Let `c_2Einteger_2Eint_of_num` : ι be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (16)$$

Let `c_2Ewords_2Edimword` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (17)$$

Let `c_2Earithmetic_2E_2D` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 22 We define `c_2Earithmetic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let `c_2Earithmetic_2EDIV` : ι be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 23 We define `c_2Ebit_2EDIV_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let `c_2Earithmetic_2EMOD` : ι be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 24 We define `c_2Ebit_2EMOD_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 26 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 27 We define c_2EfcP_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 28 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2EfcP_2EFCP$

Definition 29 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (21)$$

Let $c_2Einteger_2Eint_2REP_2CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_2REP_2CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint_2REP_2CLASS}) \quad (22)$$

Definition 30 We define $c_2Einteger_2Eint_2REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40\ t$

Let $c_2Einteger_2Etint_2neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_2neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Einteger_2Etint_2neg}) \quad (23)$$

Let $c_2Einteger_2Etint_2eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_2eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Etint_2eq}) \quad (24)$$

Let $c_2Einteger_2Eint_2ABS_2CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_2ABS_2CLASS \in (ty_2Einteger_2Eint)^{2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}} \quad (25)$$

Definition 31 We define $c_2Einteger_2Eint_2ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 32 We define $c_2Einteger_2Eint_2neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Definition 33 We define $c_2Ewords_2Eword_2msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap$

Definition 34 We define $c_2Einteger_2word_2Ew2i$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(a$

Definition 35 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 36 We define $c_2Earithmetic_2E_3C_23D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Einteger_word_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow c_2Einteger_word_2EINT_MIN \\ A_27a \in (ty_2Einteger_2Eint^{(ty_2Ebool_2Eitself \ A_27a)}) \end{aligned} \quad (26)$$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Ewords_2EINT_MIN \ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself \ A_27a)}) \quad (27)$$

Definition 37 We define $c_2Ewords_2Eword_L$ to be $\lambda A_27a : \iota.(ap \ (c_2Ewords_2En2w \ A_27a) \ (ap \ (c_2Ew$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(p \ (ap \ (ap \ c_2Earithmetic_2E_3C_3D \ V0m) \ V0m))) \quad (28)$$

Assume the following.

$$True \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg \\ (p \ V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ ((ap \ c_2Einteger_2Eint_of_num \ V0m) = (ap \ c_2Einteger_2Eint_of_num \\ V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap\ c_2Integer_2Eint_of_num\ V0m) = (ap\ c_2Integer_2Eint_of_num \\
& \quad V1n)) \Leftrightarrow (V0m = V1n)))) \wedge ((\forall V2x \in ty_2Integer_2Eint. (\forall V3y \in \\
& ty_2Integer_2Eint. (((ap\ c_2Integer_2Eint_neg\ V2x) = (ap\ c_2Integer_2Eint_neg \\
& \quad V3y)) \Leftrightarrow (V2x = V3y)))) \wedge ((\forall V4n \in ty_2Enum_2Enum. (\forall V5m \in \\
& ty_2Enum_2Enum. (((ap\ c_2Integer_2Eint_of_num\ V4n) = (ap \\
& \quad c_2Integer_2Eint_neg\ (ap\ c_2Integer_2Eint_of_num\ V5m))) \Leftrightarrow \\
& ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap\ c_2Integer_2Eint_neg \\
& (ap\ c_2Integer_2Eint_of_num\ V4n)) = (ap\ c_2Integer_2Eint_of_num \\
& \quad V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (c_2Ewords_2EINT_MIN \\
& \quad A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))\ V0n)) \wedge (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& \quad V0n)\ (ap\ (c_2Ewords_2Edimword\ A_27a)\ (c_2Ebool_2Ethe_value \\
& \quad A_27a)))) \Rightarrow ((ap\ (c_2Integer_word_2Ew2i\ A_27a)\ (ap\ (c_2Ewords_2En2w \\
& \quad A_27a)\ V0n)) = (ap\ c_2Integer_2Eint_neg\ (ap\ c_2Integer_2Eint_of_num \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2Ewords_2Edimword\ A_27a) \\
& \quad \quad (c_2Ebool_2Ethe_value\ A_27a))\ V0n))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Integer_word_2EINT_MIN \\
& \quad A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)) = (ap\ c_2Integer_2Eint_neg \\
& (ap\ c_2Integer_2Eint_of_num\ (ap\ (c_2Ewords_2EINT_MIN\ A_27a) \\
& \quad (c_2Ebool_2Ethe_value\ A_27a))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (ap\ c_2Earithmetic_2E_2D\ (\\
& \quad ap\ (c_2Ewords_2Edimword\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))) \\
& (ap\ (c_2Ewords_2EINT_MIN\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))) = \\
& (ap\ (c_2Ewords_2EINT_MIN\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (\\
& \quad ap\ (c_2Ewords_2EINT_MIN\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))) \\
& \quad (ap\ (c_2Ewords_2Edimword\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))))
\end{aligned} \tag{39}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Integer_word_2Ew2i \\
& \quad A_27a)\ (c_2Ewords_2Eword_L\ A_27a)) = (ap\ (c_2Integer_word_2EINT_MIN \\
& \quad A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))
\end{aligned}$$