

thm_2Einteger__word_2Ew2i__minus__1
 (TMVMGgX-
 gYEV8mo7ug5JoP6iNFFS7x69byeS)

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Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \tag{3}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{4}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \tag{5}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_21$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a})))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ (ty_2Einteger_2Eint\ V0a)))$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) \quad (6)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum)) \quad (7)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (8)$$

Definition 6 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 7 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg)$

Let $ty_2Efcf_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcf_2Efinite_image\ A0) \quad (9)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (10)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (11)$$

Let $c_2Efcf_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcf_2Edimindex\ A_27a \in (ty_2Enum_2Enum)^{(ty_2Ebool_2Eitself\ A_27a)} \quad (12)$$

Definition 8 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 10 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21\ 2))$

Definition 11 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (15)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E_2F_5C$

Definition 16 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \quad (16)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})(ty_2Efcp_2Ecart\ A_27a\ A_27b)) \quad (17)$$

Definition 17 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27a$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (18)$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 19 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 20 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 21 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 22 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 23 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Eboo$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 24 We define c_Ewords_Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).(ap\ (ap\ c_Ewords_Edimword\ : \iota \Rightarrow \iota)$

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ewords_Edimword\ A_27a \in (ty_Enum_Enum^{(ty_Ebool_Eitself\ A_27a)}) \quad (22)$$

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (23)$$

Definition 25 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmetic_EDIV\ : \iota)$

$$c_Earithmetic_EDIV \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (24)$$

Definition 26 We define $c_Ebit_EDIV_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

$$c_Earithmetic_EMOD \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (25)$$

Definition 27 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 28 We define c_Ebit_EBITS to be $\lambda V0h \in ty_Enum_Enum.\lambda V1l \in ty_Enum_Enum.\lambda V$

Definition 29 We define c_Ebit_EBIT to be $\lambda V0b \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum.(ap$

Definition 30 We define c_Efc_EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_Enum_Enum}).(ap$

Definition 31 We define c_Ewords_En2w to be $\lambda A_27a : \iota.\lambda V0n \in ty_Enum_Enum.(ap\ (c_Efc_EFCP$

Definition 32 We define $c_Ewords_Eword_comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).$

Definition 33 We define $c_Ewords_Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).(ap$

Definition 34 We define $c_Einteger_word_Ew2i$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).(a$

Let $c_Ewords_EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ewords_EUINT_MAX\ A_27a \in (\quad (26)$$

Definition 35 We define $c_Ewords_Eword_T$ to be $\lambda A_27a : \iota.(ap\ (c_Ewords_En2w\ A_27a)\ (ap\ (c_Ew$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow ((ap\ (c_2Einteger_word_2Ew2i\ A_{27a})\ (c_2Ewords_2Eword_T\ A_{27a})) = (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \quad (27)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow ((ap\ (c_2Ewords_2Eword_2comp\ A_{27a})\ (ap\ (c_2Ewords_2En2w\ A_{27a})\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) = (c_2Ewords_2Eword_T\ A_{27a})) \quad (28)$$

Theorem 1

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow ((ap\ (c_2Einteger_word_2Ew2i\ A_{27a})\ (ap\ (c_2Ewords_2Eword_2comp\ A_{27a})\ (ap\ (c_2Ewords_2En2w\ A_{27a})\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) = (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))$$