

# thm\_2Einteger\_\_word\_2Eword\_\_mul\_\_i2w (TMFS7V1ZHfgFRAXayJT6ep5uvfa8y53FW9r)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})_{ty\_2Einteger\_2Eint}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p\ (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_27$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a})))$

**Definition 5** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (c\_2Emin\_2E\_40\ (ty\_2Einteger\_2Eint\_REP\_CLASS\ a)))$

Let  $c\_2Einteger\_2Eint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)_{ty\_2Einteger\_2Eint\_mul})_{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})_{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \tag{5}$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (6)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \quad (7)$$

**Definition 6** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 7** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint)^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 8** We define  $c\_2Einteger\_2ENum$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint.(ap\ (c\_2Emin\_2E40\ ty\_2Enum\_2Enum))$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (9)$$

**Definition 9** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (11)$$

**Definition 10** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 11** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega)^{ty\_2Enum\_2Enum} \quad (12)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega)^{\omega} \quad (13)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (14)$$

**Definition 13** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap (ap c\_Earithmetic$

**Definition 14** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Enum\_Enum.V0x$ .

**Definition 15** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap (ap c\_Earithmetic$

Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (15)$$

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (16)$$

**Definition 16** We define  $c\_Ebit\_EDIV\_EXP$  to be  $\lambda V0x \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (17)$$

Let  $c\_Earithmetic\_EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EMOD \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (18)$$

**Definition 17** We define  $c\_Ebit\_EMOD\_EXP$  to be  $\lambda V0x \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$

**Definition 18** We define  $c\_Ebit\_EBITS$  to be  $\lambda V0h \in ty\_Enum\_Enum.\lambda V1l \in ty\_Enum\_Enum.\lambda V$

**Definition 19** We define  $c\_Ebit\_EBIT$  to be  $\lambda V0b \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum.(ap$

Let  $ty\_Efc\_Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_Efc\_Efinite\_image A0) \quad (19)$$

Let  $ty\_Ebool\_Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_Ebool\_Eitself A0) \quad (20)$$

Let  $c\_Ebool\_Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Ebool\_Ethe\_value A\_27a \in (ty\_Ebool\_Eitself A\_27a) \quad (21)$$

Let  $c\_Efc\_Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Efc\_Edimindex A\_27a \in (ty\_Enum\_Enum^{(ty\_Ebool\_Eitself A\_27a)}) \quad (22)$$

**Definition 20** We define  $c\_Ebool\_EF$  to be  $(ap (c\_Ebool\_E.21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 21** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 22** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 23** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ap (c\_2Emin\_2E\_3D\_3D\_3E V2t) c\_2Ebool\_2E\_7E)))))$

**Definition 24** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_3D\_3D\_3E V0P) c\_2Ebool\_2E\_7E))))$

**Definition 25** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Emin\_2E\_3D\_3D\_3E V0m) (ap (c\_2Emin\_2E\_3D\_3D\_3E V1n) c\_2Ebool\_2E\_7E)))$

**Definition 26** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C V0P) (ap (c\_2Emin\_2E\_3D\_3D\_3E V0P) c\_2Ebool\_2E\_7E))))$

**Definition 27** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum})) (ap (c\_2Emin\_2E\_3D\_3D\_3E V0P) c\_2Ebool\_2E\_7E)))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart A0 A1) \quad (23)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \quad (24)$$

**Definition 28** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b)$

**Definition 29** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap (c\_2Efcp\_2Efcp\_index A\_27a A\_27b) g))$

**Definition 30** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2Efcp\_2EFCP A\_27a A\_27a) V0n)$

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) \quad (25)$$

**Definition 31** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap c\_2Einteger\_2Eint\_neg V0T1)$

**Definition 32** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Ebool\_2E\_7E V2t2) (ap (c\_2Ebool\_2E\_21 2) V1t1))))))$

**Definition 33** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E\_7E V0b) c\_2Ebool\_2E\_21 2) V1n) c\_2Ebool\_2E\_7E))$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{(ty\_2Enum\_2Enum)}) \quad (26)$$

**Definition 34** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a).(ap (ap (c\_2Ewords\_2En2w A\_27a A\_27a) V0w) c\_2Ebool\_2E\_7E))$



**Definition 49** We define  $c\_2Enumeral\_2Einternal\_mult$  to be  $c\_2Earithmetic\_2E\_2A$ .

**Definition 50** We define  $c\_2Ewords\_2Eword\_add$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V$

**Definition 51** We define  $c\_2Ewords\_2Eword\_mul$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2A V1n) V0m)))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = c\_2Enum\_2E0) \vee (V1n = c\_2Enum\_2E0)))) \quad (35)$$

Assume the following.

$$True \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (42)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (43)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (44)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (47)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (49)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (50)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
& V5y\_27)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\
& A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\
& (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\
& ((ap\ (ap\ c\_2Einteger\_2Eint\_mul\ (ap\ c\_2Einteger\_2Eint\_neg\ V0x)) \\
& (ap\ c\_2Einteger\_2Eint\_neg\ V1y)) = (ap\ (ap\ c\_2Einteger\_2Eint\_mul \\
& V0x)\ V1y))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& ((ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) = (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap\ (ap\ c\_2Einteger\_2Eint\_mul\ (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V0m))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_of\_num \\
& (ap\ (ap\ c\_2Earithmic\_2E\_2A\ V0m)\ V1n))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V0n))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_neg \\
& (ap\ c\_2Einteger\_2Eint\_of\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_neg \\
& (ap\ c\_2Einteger\_2Eint\_of\_num\ V1m)))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& V1m)\ V0n))) \wedge (((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_neg \\
& (ap\ c\_2Einteger\_2Eint\_of\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V1m))) \Leftrightarrow ((\neg(V0n = c\_2Enum\_2E0)) \vee (\neg(V1m = c\_2Enum\_2E0)))) \wedge ((p \\
& (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V0n))\ (ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V1m)))) \Leftrightarrow False))))))
\end{aligned} \tag{56}$$



Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap\ c\_2Einteger\_2ENum (ap\ c\_2Einteger\_2Eint\_of\_num\ V0n)) = V0n)) \quad (57)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\ & ((ap\ (ap\ c\_2Einteger\_2Eint\_mul\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_of\_num \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))) \wedge ((\forall V2x \in ty\_2Einteger\_2Eint. \\ & (\forall V3y \in ty\_2Einteger\_2Eint. ((ap\ (ap\ c\_2Einteger\_2Eint\_mul \\ & (ap\ c\_2Einteger\_2Eint\_neg\ V2x))\ V3y) = (ap\ c\_2Einteger\_2Eint\_neg \\ & (ap\ (ap\ c\_2Einteger\_2Eint\_mul\ V2x)\ V3y)))))) \wedge ((\forall V4x \in ty\_2Einteger\_2Eint. \\ & (\forall V5y \in ty\_2Einteger\_2Eint. ((ap\ (ap\ c\_2Einteger\_2Eint\_mul \\ & V4x)\ (ap\ c\_2Einteger\_2Eint\_neg\ V5y)) = (ap\ c\_2Einteger\_2Eint\_neg \\ & (ap\ (ap\ c\_2Einteger\_2Eint\_mul\ V4x)\ V5y)))))) \wedge ((\forall V6x \in ty\_2Einteger\_2Eint. \\ & ((ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_neg\ V6x)) = \\ & V6x)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))
\end{aligned}$$

(60)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)) = (ap \\
& \quad \quad c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) \wedge ((ap \\
& \quad \quad c\_2Enumeral\_2EiDUB\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO)))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. ( \\
& \forall V2y \in ty\_2Enum\_2Enum. (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ c\_2Earithmetic\_2EZERO) \\
& \quad V0n) = c\_2Earithmetic\_2EZERO) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
& \quad \quad V0n)\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge (((ap \\
& \quad \quad (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2EBIT1\ V1x))\ (ap \\
& \quad \quad c\_2Earithmetic\_2EBIT1\ V2y)) = (ap\ (ap\ c\_2Enumeral\_2Einternal\_mult \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT1\ V1x))\ (ap\ c\_2Earithmetic\_2EBIT1\ V2y))) \wedge \\
& \quad \quad (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2EBIT1\ V1x)) \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ V2y)) = (ap\ (ap\ (c\_2Ebool\_2ELET\ ty\_2Enum\_2Enum \\
& \quad \quad ty\_2Enum\_2Enum)\ (\lambda V3n \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND \\
& \quad \quad ty\_2Enum\_2Enum)\ (ap\ c\_2Earithmetic\_2EODD\ V3n))\ (ap\ (ap\ c\_2Enumeral\_2Eexp\_help \\
& \quad \quad (ap\ c\_2Earithmetic\_2EDIV2\ V3n))\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad \quad V1x))))\ (ap\ (ap\ c\_2Enumeral\_2Einternal\_mult\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad \quad V1x))\ (ap\ c\_2Earithmetic\_2EBIT2\ V2y))))\ (ap\ c\_2Enumeral\_2Eexactlog \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ V2y)))) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ V1x))\ (ap\ c\_2Earithmetic\_2EBIT1\ V2y)) = \\
& \quad \quad (ap\ (ap\ (c\_2Ebool\_2ELET\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ (\lambda V4m \in \\
& \quad \quad ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Enum\_2Enum) \\
& \quad \quad (ap\ c\_2Earithmetic\_2EODD\ V4m))\ (ap\ (ap\ c\_2Enumeral\_2Eexp\_help \\
& \quad \quad (ap\ c\_2Earithmetic\_2EDIV2\ V4m))\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad \quad V2y))))\ (ap\ (ap\ c\_2Enumeral\_2Einternal\_mult\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& \quad \quad V1x))\ (ap\ c\_2Earithmetic\_2EBIT1\ V2y))))\ (ap\ c\_2Enumeral\_2Eexactlog \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ V1x)))) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ V1x))\ (ap\ c\_2Earithmetic\_2EBIT2\ V2y)) = \\
& \quad \quad (ap\ (ap\ (c\_2Ebool\_2ELET\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ (\lambda V5m \in \\
& \quad \quad ty\_2Enum\_2Enum. (ap\ (ap\ (c\_2Ebool\_2ELET\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \\
& \quad \quad (\lambda V6n \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Enum\_2Enum) \\
& \quad \quad (ap\ c\_2Earithmetic\_2EODD\ V5m))\ (ap\ (ap\ c\_2Enumeral\_2Eexp\_help \\
& \quad \quad (ap\ c\_2Earithmetic\_2EDIV2\ V5m))\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& \quad \quad V2y))))\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Enum\_2Enum)\ (ap\ c\_2Earithmetic\_2EODD \\
& \quad \quad V6n))\ (ap\ (ap\ c\_2Enumeral\_2Eexp\_help\ (ap\ c\_2Earithmetic\_2EDIV2 \\
& \quad \quad V6n))\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT2\ V1x))))\ \\
& \quad \quad (ap\ (ap\ c\_2Enumeral\_2Einternal\_mult\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& \quad \quad V1x))\ (ap\ c\_2Earithmetic\_2EBIT2\ V2y))))\ (ap\ c\_2Enumeral\_2Eexactlog \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ V2y))))\ (ap\ c\_2Enumeral\_2Eexactlog \\
& \quad \quad (ap\ c\_2Earithmetic\_2EBIT2\ V1x))))))))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Enumeral\_2Einternal\_mult c\_2Earithmetic\_2EZERO) \\
V0n) = c\_2Earithmetic\_2EZERO) \wedge (((ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge (((ap \\
& (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT1 \\
V0n)) V1m) = (ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Enumeral\_2EiDUB (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) V1m))) V1m))) \wedge ((ap (ap c\_2Enumeral\_2Einternal\_mult (ap \\
& c\_2Earithmetic\_2EBIT2 V0n)) V1m) = (ap c\_2Enumeral\_2EiDUB (ap \\
& c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) V1m))) V1m)))))))))
\end{aligned} \tag{63}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) c\_2Enum\_2E0)))) \tag{64}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{65}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{66}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{67}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{68}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{74}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{79}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0m \in \text{ty\_2Enum\_2Enum}. ( \\
& \forall V1n \in \text{ty\_2Enum\_2Enum}. (((ap \ (c\_2Ewords\_2En2w \ A\_27a) \ V0m) = \\
& (ap \ (c\_2Ewords\_2En2w \ A\_27a) \ V1n)) \Leftrightarrow ((ap \ (ap \ c\_2Earithmetic\_2EMOD \\
& \ V0m) \ (ap \ (c\_2Ewords\_2Edimword \ A\_27a) \ (c\_2Ebool\_2Ethe\_value \\
& \ A\_27a))) = (ap \ (ap \ c\_2Earithmetic\_2EMOD \ V1n) \ (ap \ (c\_2Ewords\_2Edimword \\
& \ A\_27a) \ (c\_2Ebool\_2Ethe\_value \ A\_27a))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c.2Ewords\_2Ew2n\ A.27a)\ (ap\ (c.2Ewords\_2En2w\ A.27a)\ c.2Enum\_2E0)) = c.2Enum\_2E0) \quad (81)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty\_2EfcP\_2Ecart\ 2\ A.27a).(((ap\ (c.2Ewords\_2Ew2n\ A.27a)\ V0w) = c.2Enum\_2E0) \Leftrightarrow (V0w = (ap\ (c.2Ewords\_2En2w\ A.27a)\ c.2Enum\_2E0)))) \quad (82)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0v \in (ty\_2EfcP\_2Ecart\ 2\ A.27a).(\forall V1w \in (ty\_2EfcP\_2Ecart\ 2\ A.27a).(\forall V2x \in (ty\_2EfcP\_2Ecart\ 2\ A.27a).(((ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V0v)\ (ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V1w)\ V2x)) = (ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ (ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V0v)\ V1w))\ V2x)))))) \quad (83)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0v \in (ty\_2EfcP\_2Ecart\ 2\ A.27a).(\forall V1w \in (ty\_2EfcP\_2Ecart\ 2\ A.27a).((ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V0v)\ V1w) = (ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V1w)\ V0v)))) \quad (84)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0v \in (ty\_2EfcP\_2Ecart\ 2\ A.27a).(\forall V1w \in (ty\_2EfcP\_2Ecart\ 2\ A.27a).(((ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ (ap\ (c.2Ewords\_2En2w\ A.27a)\ c.2Enum\_2E0))\ V0v) = (ap\ (c.2Ewords\_2En2w\ A.27a)\ c.2Enum\_2E0)) \wedge (((ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ (ap\ (c.2Ewords\_2En2w\ A.27a)\ c.2Enum\_2E0))\ (ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ (ap\ c.2Earithmetic\_2ENUMERAL\ (ap\ c.2Earithmetic\_2EBIT1\ c.2Earithmetic\_2EZERO))))\ V0v) = V0v) \wedge (((ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V0v)\ (ap\ (c.2Ewords\_2En2w\ A.27a)\ (ap\ c.2Earithmetic\_2ENUMERAL\ (ap\ c.2Earithmetic\_2EBIT1\ c.2Earithmetic\_2EZERO)))) = V0v) \wedge (((ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ (ap\ (ap\ (c.2Ewords\_2Eword\_add\ A.27a)\ V0v)\ (ap\ (c.2Ewords\_2En2w\ A.27a)\ (ap\ c.2Earithmetic\_2ENUMERAL\ (ap\ c.2Earithmetic\_2EBIT1\ c.2Earithmetic\_2EZERO))))\ V1w) = (ap\ (ap\ (c.2Ewords\_2Eword\_add\ A.27a)\ (ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V0v)\ V1w))\ V1w)) \wedge (((ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V0v)\ (ap\ (ap\ (c.2Ewords\_2Eword\_add\ A.27a)\ V1w)\ (ap\ (c.2Ewords\_2En2w\ A.27a)\ (ap\ c.2Earithmetic\_2ENUMERAL\ (ap\ c.2Earithmetic\_2EBIT1\ c.2Earithmetic\_2EZERO)))) = (ap\ (ap\ (c.2Ewords\_2Eword\_add\ A.27a)\ V0v)\ (ap\ (ap\ (c.2Ewords\_2Eword\_mul\ A.27a)\ V0v)\ V1w)))))) \quad (85)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0v \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).(((ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ V0v) = (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0))) \Leftrightarrow (V0v = (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0)))) \quad (86)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).((ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ V0w) = (ap\ (ap\ (c\_2Ewords\_2Eword\_2mul\ A\_27a)\ (ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ V0w)))) \quad (87)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ewords\_2Eword\_2mul\ A\_27a)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ V0m))\ (ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ V1n)))) = (ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))) \wedge (\forall V2m \in ty\_2Enum\_2Enum.(\forall V3n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ewords\_2Eword\_2mul\ A\_27b)\ (ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27b)\ (ap\ (c\_2Ewords\_2En2w\ A\_27b)\ V2m))))\ (ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27b)\ (ap\ (c\_2Ewords\_2En2w\ A\_27b)\ V3n)))) = (ap\ (c\_2Ewords\_2En2w\ A\_27b)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V2m)\ V3n)))))) \quad (88)$$

### Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).(\forall V1b \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).((ap\ (c\_2Einteger\_2word\_2Ei2w\ A\_27a)\ (ap\ (ap\ c\_2Einteger\_2Eint\_2mul\ (ap\ (c\_2Einteger\_2word\_2Ew2i\ A\_27a)\ V0a))\ (ap\ (c\_2Einteger\_2word\_2Ew2i\ A\_27a)\ V1b)))) = (ap\ (c\_2Ewords\_2Eword\_2mul\ A\_27a)\ V0a)\ V1b))))))$$