thm_2Eintegral_2EDINT__EQ (TM-MZaSX3MsZ6JvWEW9MKqGrmswJerS7VPN1)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (\mathsf{c_2Emin_2E_40}\ A$

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 6 *We define* c_2Ecombin_2ES *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}$

Definition 7 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ (\mathsf{c_2Ecombin_2ES}\ A_27a\ (A_27a^{A_27a})\ A_2$

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27a}$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{1}$$

Let $c_2Epair_2EABS__prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS__prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})}) \tag{2}$$

Definition 11 *We define* c_2Epair_2E_2C *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Etransc_2EDint : \iota$ be given. Assume the following.

$$c_2Etransc_2EDint \in (((2^{ty_2Erealax_2Ereal})^{(ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})})^{(ty_2Epair_2Eprod\ ty_2Erealax} \tag{4}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{5}$$

Let $c_2Erealax_2Ereal__REP__CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal__REP__CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax} \tag{6}$$

Definition 12 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (\mathsf{c_2Emin_2E_40}\ (t$

Let $c_2Erealax_2Etreal__lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal__lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Eh} \tag{7}$$

Definition 13 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_$

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c_2Emin_2E_3D_3D_3E}\ V0t)\ \mathsf{c_2Ebool_2E}$

Definition 16 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Er$

Definition 17 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{8}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{14}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{19}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\exists V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \vee (p\ V1Q)) \Leftrightarrow (\exists V3x \in A_27a.((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \tag{20}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p\ V0P) \vee (\exists V2x \in A_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \tag{21}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee(\\ (p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{22}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\vee(p\ V1B))\Leftrightarrow((p\ V1B)\vee\\ (p\ V0A))))) \tag{23}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(\\ p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{24}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow\forall A_27b.nonempty\ A_27b\Rightarrow(\\ \forall V0P \in ((2^{A_27b})^{A_27a}).((\forall V1x \in A_27a.(\exists V2y \in\\ A_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y))))\Leftrightarrow(\exists V3f \in (A_27b^{A_27a}).(\\ \forall V4x \in A_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \tag{25}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow(\forall V0x \in A_27a.((ap\ (\mathsf{c_2Ecombin_2EI}\\ A_27a)\ V0x) = V0x)) \tag{26}$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in\\ (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2a \in ty_2Erealax_2Ereal.\\ (\forall V3b \in ty_2Erealax_2Ereal.(\forall V4i \in ty_2Erealax_2Ereal.\\ (\forall V5j \in ty_2Erealax_2Ereal.(((p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\\ V2a)\ V3b))\wedge((p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b))\ V0f)\ V4i))\wedge\\ ((p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ ty_2Erealax_2Ereal\\ ty_2Erealax_2Ereal)\ V2a)\ V3b))\ V1g)\ V5j))\wedge(\forall V6x \in ty_2Erealax_2Ereal.\\ (((p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ V2a)\ V6x))\wedge(p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\\ V6x)\ V3b)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ (ap\ V0f\ V6x))\ (ap\ V1g\\ V6x)))))))\Rightarrow(p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ V4i)\ V5j))))))))) \tag{27}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.\\ (((p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\\ V1y)\ V0x)))\Leftrightarrow(V0x = V1y)))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{30}$$

Assume the following.

$$\begin{gathered}(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \end{gathered} \tag{31}$$

Assume the following.

$$\begin{gathered}(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \end{gathered} \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{33}$$

Assume the following.

$$\begin{gathered}(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(\\ p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{gathered} \tag{34}$$

Assume the following.

$$\begin{gathered}(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{gathered} \tag{35}$$

Assume the following.

$$\begin{gathered}(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{gathered} \tag{36}$$

Assume the following.

$$\begin{gathered}(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\\ \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{gathered} \tag{37}$$

Assume the following.

$$\begin{gathered}(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee \\ (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \end{gathered} \tag{38}$$

Theorem 1

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2a \in ty_2Erealax_2Ereal.(\forall V3b \in ty_2Erealax_2Ereal.(\forall V4i \in ty_2Erealax_2Ereal.(\forall V5j \in ty_2Erealax_2Ereal.(((p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ V2a)\ V3b))\wedge((p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b))\ V0f)\ V4i))\wedge((p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b))\ V1g)\ V5j))\wedge(\forall V6x \in ty_2Erealax_2Ereal.(((p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ V2a)\ V6x))\wedge(p\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__lte}\ V6x)\ V3b)))\Rightarrow((ap\ V0f\ V6x) = (ap\ V1g\ V6x)))))))\Rightarrow(V4i = V5j))))))))$$