

# thm\_2Eintegral\_2EDINT\_LE (TMSupt1xrfam54CL1xiorc4u4UgyGCND5Fr)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.V0x)$

**Definition 4** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A.\lambda c^{A.\lambda b})^{A.\lambda a}))$

**Definition 5** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A.\lambda a : \iota.(ap (ap (c\_2Ecombin\_2ES A.\lambda a (A.\lambda a^{A.\lambda a})) A.\lambda a))$

**Definition 6** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A.\lambda a})).(ap V0P (ap (c\_2Emin\_2E\_40 A.\lambda a)))$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Etransc\_2EDint : \iota$  be given. Assume the following.

$$c\_2Etransc\_2EDint \in (((2^{ty\_2Erealx\_2Ereal})^{(ty\_2Erealx\_2Ereal)^{ty\_2Erealx\_2Ereal}})^{(ty\_2Epair\_2Eprod\ ty\_2Erealx\_2Ereal)}) \tag{3}$$

Let  $c\_2Eintegral\_2Eintegral : \iota$  be given. Assume the following.

$$c\_2Eintegral\_2Eintegral \in ((ty\_2Erealx\_2Ereal^{(ty\_2Erealx\_2Ereal)^{ty\_2Erealx\_2Ereal}})^{(ty\_2Epair\_2Eprod\ ty\_2Erealx\_2Ereal)}) \tag{4}$$

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .



Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (15) \end{aligned}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \quad (16) \end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \quad (19) \end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & 2. (((\forall V2x \in A\_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q))) \Leftrightarrow (\forall V3x \in \\ & A\_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & 2. (((\exists V2x \in A\_27a. (p\ (ap\ V0P\ V2x))) \vee (p\ V1Q))) \Leftrightarrow (\exists V3x \in \\ & A\_27a. ((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in ( \\ & 2^{A\_27a}). (((p\ V0P) \vee (\exists V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ & A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & 2. ((\exists V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A\_27a. (p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in ( \\ & 2^{A\_27a}). (((\exists V2x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ & V0P) \wedge (\exists V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in ( \\ & 2^{A\_27a}). (((\forall V2x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ & V0P) \vee (\forall V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ( \\ & (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ & (p\ V0A)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg( \\ & p\ V0A) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).( \\ \forall V4x \in A.27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((ap\ (c.2Ecombin.2EI\ A.27a)\ V0x) = V0x)) \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0a \in ty.2Erealax.2Ereal.(\forall V1b \in ty.2Erealax.2Ereal. \\ (\forall V2f \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}).((p\ ( \\ ap\ (ap\ c.2Eintegral.2Eintegrable\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal \\ ty.2Erealax.2Ereal)\ V0a)\ V1b))\ V2f))) \Leftrightarrow (\exists V3i \in ty.2Erealax.2Ereal. \\ (p\ (ap\ (ap\ (ap\ c.2Etrasc.2EDint\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal \\ ty.2Erealax.2Ereal)\ V0a)\ V1b))\ V2f)\ V3i)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0f \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}).(\forall V1a \in \\ ty.2Erealax.2Ereal.(\forall V2b \in ty.2Erealax.2Ereal.(\forall V3i \in \\ ty.2Erealax.2Ereal.(((p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ V1a)\ V2b)) \wedge \\ (p\ (ap\ (ap\ (ap\ c.2Etrasc.2EDint\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal \\ ty.2Erealax.2Ereal)\ V1a)\ V2b))\ V0f)\ V3i)))) \Rightarrow ((ap\ (ap\ c.2Eintegral.2Eintegral \\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal) \\ V1a)\ V2b))\ V0f) = V3i)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0f \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}).(\forall V1g \in \\ (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}).(\forall V2a \in ty.2Erealax.2Ereal. \\ (\forall V3b \in ty.2Erealax.2Ereal.(\forall V4i \in A.27a.(\forall V5j \in \\ A.27b.(((p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ V2a)\ V3b)) \wedge ((p\ (ap\ (ap \\ c.2Eintegral.2Eintegrable\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal \\ ty.2Erealax.2Ereal)\ V2a)\ V3b))\ V0f)) \wedge ((p\ (ap\ (ap\ c.2Eintegral.2Eintegrable \\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal) \\ V2a)\ V3b))\ V1g)) \wedge (\forall V6x \in ty.2Erealax.2Ereal.(((p\ (ap\ (ap \\ c.2Ereal.2Ereal\_lte\ V2a)\ V6x)) \wedge (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte \\ V6x)\ V3b)))) \Rightarrow (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ (ap\ V0f\ V6x))\ (ap\ V1g \\ V6x)))))) \Rightarrow (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ (ap\ (ap\ c.2Eintegral.2Eintegral \\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal) \\ V2a)\ V3b))\ V0f))\ (ap\ (ap\ c.2Eintegral.2Eintegral\ (ap\ (ap\ (c.2Epair.2E.2C \\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal)\ V2a)\ V3b))\ V1g)))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (50)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\ & (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2a \in ty\_2Erealax\_2Ereal. \\ & (\forall V3b \in ty\_2Erealax\_2Ereal.(\forall V4i \in ty\_2Erealax\_2Ereal. \\ & (\forall V5j \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte \\ & V2a) V3b)) \wedge ((p (ap (ap (ap c\_2Etransc\_2EDint (ap (ap (c\_2Epair\_2E\_2C \\ & ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V2a) V3b)) V0f) V4i)) \wedge \\ & ((p (ap (ap (ap c\_2Etransc\_2EDint (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) V2a) V3b)) V1g) V5j)) \wedge (\forall V6x \in ty\_2Erealax\_2Ereal. \\ & (((p (ap (ap c\_2Ereal\_2Ereal\_lte V2a) V6x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\ & V6x) V3b)))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap V0f V6x)) (ap V1g \\ & V6x)))))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte V4i) V5j)))))) \end{aligned}$$