

thm_2Eintegral_2EDINT__LINEAR (TMX6GjN2jkh7RFTinQsnKTnubT8USDJ8ZTS)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_7E$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal_REP_CLASS a)))$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) (5)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} (7)$$

Definition 9 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 10 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) (8)$$

Definition 12 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b}A_27a})} (9)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ x\ y)$

Let $c_2Etransc_2EDint : \iota$ be given. Assume the following.

$$c_2Etransc_2EDint \in (((2^{(ty_2Erealax_2Ereal)})(ty_2Erealax_2Ereal)^{(ty_2Erealax_2Ereal)})(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal)) (10)$$

Assume the following.

$$True (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2a \in ty_2Erealax_2Ereal. \\
& (\forall V3b \in ty_2Erealax_2Ereal.(\forall V4i \in ty_2Erealax_2Ereal. \\
& (\forall V5j \in ty_2Erealax_2Ereal.(((p (ap (ap (ap c_2Etransc_2EDint \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
V2a) V3b)) V0f) V4i)) \wedge (p (ap (ap (ap c_2Etransc_2EDint (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V2a) V3b)) V1g) V5j)))) \Rightarrow \\
& (p (ap (ap (ap c_2Etransc_2EDint (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V2a) V3b)) (\lambda V6x \in ty_2Erealax_2Ereal. \\
& (ap (ap c_2Erealax_2Ereal_add (ap V0f V6x)) (ap V1g V6x)))) (ap \\
& (ap c_2Erealax_2Ereal_add V4i) V5j)))))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1a \in \\
& ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.(\forall V3c \in \\
& ty_2Erealax_2Ereal.(\forall V4i \in ty_2Erealax_2Ereal.((p (ap \\
& (ap (ap c_2Etransc_2EDint (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V1a) V2b)) V0f) V4i)) \Rightarrow (p (ap (ap (ap c_2Etransc_2EDint \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V1a) V2b)) (\lambda V5x \in ty_2Erealax_2Ereal.(ap (ap c_2Erealax_2Ereal_mul \\
& V3c) (ap V0f V5x)))) (ap (ap c_2Erealax_2Ereal_mul V3c) V4i)))))))))
\end{aligned} \tag{14}$$

Theorem 1

$$\begin{aligned}
& (\forall V0m \in ty_2Erealax_2Ereal.(\forall V1n \in ty_2Erealax_2Ereal. \\
& (\forall V2f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V3g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V4a \in ty_2Erealax_2Ereal. \\
& (\forall V5b \in ty_2Erealax_2Ereal.(\forall V6i \in ty_2Erealax_2Ereal. \\
& (\forall V7j \in ty_2Erealax_2Ereal.(((p (ap (ap (ap c_2Etransc_2EDint \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
V4a) V5b)) V2f) V6i)) \wedge (p (ap (ap (ap c_2Etransc_2EDint (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V4a) V5b)) V3g) V7j)))) \Rightarrow \\
& (p (ap (ap (ap c_2Etransc_2EDint (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V4a) V5b)) (\lambda V8x \in ty_2Erealax_2Ereal. \\
& (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_mul \\
V0m) (ap V2f V8x))) (ap (ap c_2Erealax_2Ereal_mul V1n) (ap V3g V8x)))))) \\
& (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_mul \\
V0m) V6i)) (ap (ap c_2Erealax_2Ereal_mul V1n) V7j)))))))))
\end{aligned}$$