

# thm\_2Eintegral\_2EDINT\_\_NEG (TMG8rGew15AHhqPFB3UherYrznabHBjQbEu)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax\_2Ereal) \tag{4}$$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal) \quad (5)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_2ABS\_2CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_2ABS\_2CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}}) \quad (7)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_2ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_2add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (8)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_2neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (9)$$

Let  $c\_2Enum\_2EREP\_2num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_2num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_2REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_2REP \in (\omega^{\omega^{omega}}) \quad (11)$$

Let  $c\_2Enum\_2EABS\_2num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_2num \in (ty\_2Enum\_2Enum^{\omega^{omega}}) \quad (12)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_2num$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal) \quad (13)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_2mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$



Let  $c\_2Etransc\_2Efine : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Efine \in ((2^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})}))) \quad (19)$$

Let  $c\_2Etransc\_2Etdiv : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Etdiv \in ((2^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})}))) \quad (20)$$

**Definition 27** We define  $c\_2Etransc\_2Egauge$  to be  $\lambda V0E \in (2^{ty\_2Erealax\_2Ereal}). \lambda V1g \in (ty\_2Erealax\_2Ereal$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b} A\_27a)}) \end{aligned} \quad (21)$$

**Definition 28** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2$

Let  $c\_2Etransc\_2EDint : \iota$  be given. Assume the following.

$$c\_2Etransc\_2EDint \in (((2^{ty\_2Erealax\_2Ereal})^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})})^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal)}) \quad (22)$$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ p V0t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (28)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \Rightarrow (29)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Ereal\_sub (ap c\_2Erealax\_2Ereal\_neg V0x)) V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)))))) (30)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap c\_2Ereal\_2Eabs (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Eabs V0x))) (31)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2d \in ty\_2Enum\_2Enum.((ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) V1n) V2d)) (\lambda V3n \in ty\_2Enum\_2Enum.(ap c\_2Erealax\_2Ereal\_neg (ap V0f V3n)))) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) V1n) V2d)) V0f)))))) (32)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)))))) (33)$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). ((ap (ap c\_2Etransc\_2Ersum (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum} ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V0D) V1p)) V2f) = (ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) c\_2Enum\_2E0) (ap c\_2Etransc\_2Edsize V0D))) (\lambda V3n \in ty\_2Enum\_2Enum. (ap (ap c\_2Erealax\_2Ereal\_mul (ap V2f (ap V1p V3n))) (ap (ap c\_2Ereal\_2Ereal\_sub (ap V0D (ap c\_2Enum\_2ESUC V3n))) (ap V0D V3n))))))))) (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1b \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V3k \in \\
& \quad ty\_2Erealax\_2Ereal. ((p (ap (ap (ap c\_2Etransc\_2EDint (ap (ap ( \\
& \quad c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0a) \\
& \quad V1b)) V2f) V3k))) \Leftrightarrow (\forall V4e \in ty\_2Erealax\_2Ereal. ((p (ap (ap \\
& \quad c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0) \\
& \quad V4e))) \Rightarrow (\exists V5g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap c\_2Etransc\_2Egauge (\lambda V6x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Ereal\_2Ereal\_lte V0a) V6x)) \\
& \quad (ap (ap c\_2Ereal\_2Ereal\_lte V6x) V1b)))) V5g)) \wedge (\forall V7D \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V8p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& \quad (((p (ap (ap c\_2Etransc\_2Etdiv (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V0a) V1b)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum} \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V7D) V8p))) \wedge (p (ap (ap c\_2Etransc\_2Efine \\
& \quad V5g) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum} \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V7D) V8p)))))) \Rightarrow (p (ap (ap \\
& \quad c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub \\
& \quad (ap (ap c\_2Etransc\_2Ersum (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum} \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V7D) V8p)) V2f)) V3k))) \\
& \quad V4e))))))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1a \in \\
& \quad ty\_2Erealax\_2Ereal. (\forall V2b \in ty\_2Erealax\_2Ereal. (\forall V3i \in \\
& \quad ty\_2Erealax\_2Ereal. ((p (ap (ap (ap c\_2Etransc\_2EDint (ap (ap ( \\
& \quad c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V1a) \\
& \quad V2b)) V0f) V3i))) \Rightarrow (p (ap (ap (ap c\_2Etransc\_2EDint (ap (ap (c\_2Epair\_2E\_2C \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V1a) V2b)) (\lambda V4x \in \\
& \quad ty\_2Erealax\_2Ereal. (ap c\_2Erealax\_2Ereal\_neg (ap V0f V4x)))) \\
& \quad (ap c\_2Erealax\_2Ereal\_neg V3i)))))))))
\end{aligned}$$