# thm_2Eintegral_2EDIVISION__APPEND__STRONG (TMRcX64vLSfksykU6MA1hiSHBfnsjZNvWGf)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

**Definition 3** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 4** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

**Definition 5** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 6** *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh}\tag{5}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh}\tag{6}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh}}\tag{7}$$

**Definition 7** *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

**Definition 8** *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum\tag{8}$$

Let $c\_2Etransc\_2Ersum : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Ersum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})})^{(ty\_2Epair\_2Eprod\ (ty\_2Erea}\tag{9}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})\tag{10}$$

**Definition 9** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 10** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 11** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

**Definition 12** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum})\tag{11}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega})\tag{12}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega})\tag{13}$$

**Definition 13** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

**Definition 14** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

**Definition 15** *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 16** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

**Definition 17** *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{14}$$

**Definition 18** *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

**Definition 19** *We define* c_2Etransc_2Edsize *to be* $\lambda V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap\ (\mathsf{c\_2}$

**Definition 20** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Etransc\_2Efine : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Efine \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enu} \tag{15}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{16}$$

**Definition 21** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Etransc\_2Etdiv : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Etdiv \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enu} \tag{17}$$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{22}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{25}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B))))))) \tag{26}$$

Assume the following.

$$
(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.
(\forall V2c \in ty\_2Erealax\_2Ereal.(\forall V3g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).
(\forall V4d1 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V5p1 \in
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V6d2 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).
(\forall V7p2 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(((p\ (ap
(ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal
ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V4d1)\ V5p1)))\wedge((p\ (ap\ (
ap\ c\_2Etransc\_2Efine\ V3g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V4d1)\ V5p1)))\wedge((p\ (ap\ (
ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal
ty\_2Erealax\_2Ereal)\ V1b)\ V2c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V6d2)\ V7p2)))\wedge(p\ (ap\ (ap
c\_2Etransc\_2Efine\ V3g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V6d2)\ V7p2))))))\Rightarrow((p\ (
ap\ (ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal
ty\_2Erealax\_2Ereal)\ V0a)\ V2c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ (\lambda V8n \in ty\_2Enum\_2Enum.
(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}
V8n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V4d1)))\ (ap\ V4d1\ V8n))\ (ap\ V6d2\ (ap\ (
ap\ c\_2Earithmetic\_2E\_2D\ V8n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V4d1))))))
(\lambda V9n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)
(ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V9n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V4d1)))
(ap\ V5p1\ V9n))\ (ap\ V7p2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}
V4d1))))))))\wedge((p\ (ap\ (ap\ c\_2Etransc\_2Efine\ V3g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))
(\lambda V10n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)
(ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V10n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V4d1)))
(ap\ V4d1\ V10n))\ (ap\ V6d2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V10n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}
V4d1))))))\ (\lambda V11n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}
ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V11n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}
V4d1)))\ (ap\ V5p1\ V11n))\ (ap\ V7p2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V11n)
(ap\ \mathsf{c\_2Etransc\_2Edsize}\ V4d1)))))))))\wedge(\forall V12f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).
((ap\ (ap\ c\_2Etransc\_2Ersum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ (\lambda V13n \in ty\_2Enum\_2Enum.
(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}
V13n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V4d1)))\ (ap\ V4d1\ V13n))\ (ap\ V6d2\ (ap
(ap\ c\_2Earithmetic\_2E\_2D\ V13n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V4d1))))))
(\lambda V14n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)
(ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V14n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V4d1)))
(ap\ V5p1\ V14n))\ (ap\ V7p2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V14n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}
V4d1)))))))\ V12f) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ c\_2Etransc\_2Ersum
(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V4d1)\ V5p1))\ V12f))\ (ap
(ap\ c\_2Etransc\_2Ersum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V6d2)\ V7p2))\ V12f)))))))))))))))
\tag{27}
$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( (p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg( p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( (p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( (p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{35}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( (p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge(( \neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{37}$$

**Theorem 1**

$$
\begin{gathered}
(\forall V0g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in \\
ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(\forall V3c \in \\
ty\_2Erealax\_2Ereal.(\forall V4D1 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
(\forall V5p1 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V6D2 \in \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V7p2 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
(((p\ (ap\ (ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V4D1)\ V5p1)))\wedge((p\ (ap\ ( \\
ap\ c\_2Etransc\_2Efine\ V0g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V4D1)\ V5p1)))\wedge((p\ (ap\ ( \\
ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal)\ V2b)\ V3c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V6D2)\ V7p2)))\wedge(p\ (ap\ (ap \\
c\_2Etransc\_2Efine\ V0g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V6D2)\ V7p2))))))\Rightarrow(\exists V8D \in \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\exists V9p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
((p\ (ap\ (ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal)\ V1a)\ V3c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V8D)\ V9p)))\wedge((p\ (ap\ (ap \\
c\_2Etransc\_2Efine\ V0g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V8D)\ V9p)))\wedge(\forall V10f \in \\
(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).((ap\ (ap\ c\_2Etransc\_2Ersum \\
(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V8D)\ V9p))\ V10f) = (ap\ (ap \\
\mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ c\_2Etransc\_2Ersum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C} \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) \\
V4D1)\ V5p1))\ V10f))\ (ap\ (ap\ c\_2Etransc\_2Ersum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C} \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) \\
V6D2)\ V7p2))\ V10f)))))))))))))))))
\end{gathered}
$$