

# thm\_2Eintegral\_2EDIVISION\_\_MONO\_\_LE (TMHBFh7dwT9jN41eLMqsugS7VZQ96uNAiGz)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{2}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (P)) (V0t) (V1t)))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega}) \tag{4}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ ).

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (7)$$

**Definition 14** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (8)$$

Let  $c\_2Etransc\_2Edivision : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Edivision \in ((2^{(ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealx\_2Ereal\ ty\_2Erealx\_2Ereal)}) \quad (9)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (11)$$

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Erealx\_2Ereal}) \quad (12)$$

**Definition 16** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ ($

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 18** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & ((ap (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ & ap\ c\_2Earithmetic\_2E\_2B\ V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap\ c\_2Earithmetic\_2E\_2B \\ & (ap\ c\_2Enum\_2ESUC\ V0m)) V1n) = (ap\ c\_2Enum\_2ESUC\ (ap (ap\ c\_2Earithmetic\_2E\_2B \\ & V0m) V1n))) \wedge ((ap (ap\ c\_2Earithmetic\_2E\_2B\ V0m) (ap\ c\_2Enum\_2ESUC \\ & V1n)) = (ap\ c\_2Enum\_2ESUC\ (ap (ap\ c\_2Earithmetic\_2E\_2B\ V0m) V1n))))))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (p (ap (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0m) V1n)) \Leftrightarrow (\exists V2p \in ty\_2Enum\_2Enum. \\ & (V1n = (ap (ap\ c\_2Earithmetic\_2E\_2B\ V0m) V2p)))))) \end{aligned} \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ & 2. ((\forall V2x \in A.27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A.27a. (p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (2^{A.27a}). (\forall V1v \in \\ & A.27a. ((\forall V2x \in A.27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ ( \\ & ap\ V0f\ V1v)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0d \in (ty.2Erealax.2Ereal^{ty.2Enum.2Enum}). (\forall V1a \in \\ & ty.2Erealax.2Ereal. (\forall V2b \in ty.2Erealax.2Ereal. ((p\ (ap \\ & (ap\ c.2Etransc.2Edivision\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal \\ & ty.2Erealax.2Ereal)\ V1a)\ V2b))\ V0d)) \Rightarrow (\forall V3n \in ty.2Enum.2Enum. \\ & (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ (ap\ V0d\ V3n))\ (ap\ V0d\ (ap\ c.2Enum.2ESUC \\ & V3n)))))))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty.2Enum.2Enum}). (((p\ (ap\ V0P\ c.2Enum.2E0)) \wedge \\ & (\forall V1n \in ty.2Enum.2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum.2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum. (p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (27)$$

Assume the following.

$$(\forall V0x \in ty.2Erealax.2Ereal. (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ V0x)\ V0x))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealax.2Ereal. (\forall V1y \in ty.2Erealax.2Ereal. \\ & (\forall V2z \in ty.2Erealax.2Ereal. (((p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte \\ & V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ V1y)\ V2z))) \Rightarrow (p\ (ap\ ( \\ & ap\ c.2Ereal.2Ereal\_lte\ V0x)\ V2z)))))) \end{aligned} \quad (29)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0d \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in \\ & ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p (ap \\ & (ap c\_2Etransc\_2Edivision (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) V1a) V2b)) V0d)) \Rightarrow (\forall V3m \in ty\_2Enum\_2Enum. \\ & (\forall V4n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & V3m) V4n)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap V0d V3m)) (ap V0d \\ & V4n)))))))))) \end{aligned}$$