

# thm\_2Eintegral\_2EGAUGE\_\_MIN\_\_FINITE (TMWFKf2hdSwJVF4Ve4DokGkF9XJjW8Wy2d2)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (ap c\_2Enum\_2EREP\_num m))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_2E\_40$

**Definition 11** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{6}$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \tag{7}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{8}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{9}$$

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal}) \tag{10}$$

**Definition 15** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal. (ap\ (c\_Emin\_2E\_40\ ($

Let  $c\_2Erealx\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \tag{11}$$

**Definition 16** We define  $c\_2Erealx\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal. \lambda V1T2 \in ty\_2Erealx\_2E$

**Definition 17** We define  $c\_2Etransc\_2Egauge$  to be  $\lambda V0E \in (2^{ty\_2Erealx\_2Ereal}). \lambda V1g \in (ty\_2Erealx\_2E$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{12}$$

**Definition 18** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let  $c\_2Etransc\_2Efine : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Efine \in ((2^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})}))) \quad (13)$$

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & V0n) c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \wedge (\forall V1m \in ty\_2Enum\_2Enum. \\ & (\forall V2n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & V1m) (ap c\_2Enum\_2ESUC V2n))) \Leftrightarrow ((V1m = (ap c\_2Enum\_2ESUC V2n)) \vee \\ & (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V2n))))))) \end{aligned} \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge \\ & ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \end{aligned} \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in \\
& A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in \\
& A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ (2^{A.27a}).((\forall V2x \in A.27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\forall V3x \in A.27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A.27a.(p\ ( \\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((p\ V0P) \vee (\exists V2x \in A.27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ A.27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((\exists V2x \in A.27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A.27a.(p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ( \\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg( \\ p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ (p\ V1B)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ 2.(((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (2^{A-27a}).(\forall V1v \in \\ A.27a.((\forall V2x \in A.27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ ( \\ ap\ V0f\ V1v)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0P \in ((2^{A-27b})^{A-27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A-27a}).( \\ \forall V4x \in A.27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in (2^{ty.2Enum.2Enum}).(((p\ (ap\ V0P\ c.2Enum.2E0)) \wedge \\ (\forall V1n \in ty.2Enum.2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c.2Enum.2ESUC \\ V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (45)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (48)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ p\ V2r) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0E \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1g1 \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& (\forall V2g2 \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). ((( \\
& p \ (ap \ (ap \ c\_2Etransc\_2Egauge \ V0E) \ V1g1)) \wedge (p \ (ap \ (ap \ c\_2Etransc\_2Egauge \\
& V0E) \ V2g2))) \Rightarrow (p \ (ap \ (ap \ c\_2Etransc\_2Egauge \ V0E) \ (\lambda V3x \in ty\_2Erealax\_2Ereal. \\
& (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ ty\_2Erealax\_2Ereal) \ (ap \ (ap \ c\_2Erealax\_2Ereal\_lt \\
& (ap \ V1g1 \ V3x)) \ (ap \ V2g2 \ V3x))) \ (ap \ V1g1 \ V3x)) \ (ap \ V2g2 \ V3x))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0g1 \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g2 \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& (\forall V3p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p (ap (ap \\
& c\_2Etransc\_2Efine (\lambda V4x \in ty\_2Erealax\_2Ereal.(ap (ap (ap ( \\
& c\_2Ebool\_2ECOND ty\_2Erealax\_2Ereal) (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap V0g1 V4x)) (ap V1g2 V4x))) (ap V0g1 V4x)) (ap V1g2 V4x)))) (ap ( \\
ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) \\
V2D) V3p))) \Rightarrow ((p (ap (ap c\_2Etransc\_2Efine V0g1) (ap (ap (c\_2Epair\_2E\_2C \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) \\
V2D) V3p))) \wedge (p (ap (ap c\_2Etransc\_2Efine V1g2) (ap (ap (c\_2Epair\_2E\_2C \\
(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) \\
V2D) V3p)))))))))
\end{aligned} \tag{60}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1gs \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})^{ty\_2Enum\_2Enum}). \\
& (\forall V2n \in ty\_2Enum\_2Enum.((\forall V3m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V3m) V2n)) \Rightarrow (p (ap (ap c\_2Etransc\_2Egauge \\
& V0s) (ap V1gs V3m)))))) \Rightarrow (\exists V4g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap c\_2Etransc\_2Egauge V0s) V4g)) \wedge (\forall V5d \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& (\forall V6p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p (ap (ap \\
& c\_2Etransc\_2Efine V4g) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V5d) V6p))) \Rightarrow (\forall V7m \in \\
& ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V7m) V2n)) \Rightarrow \\
& (p (ap (ap c\_2Etransc\_2Efine (ap V1gs V7m)) (ap (ap (c\_2Epair\_2E\_2C \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) \\
& V5d) V6p)))))))))
\end{aligned}$$