

thm_2Eintegral_2EINTEGRABLE__ADD (TM-MJBmE9PjgqY9ELkECuxScKQoTEGA4Q5Km)

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Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_21$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A.^{27a}}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A.^{27a}}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty_2Erealax_2Ereal_REP_CLASS\ a))\ (ty_2Erealax_2Ereal_REP_CLASS\ a))$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{5}$$

Definition 6 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 7 We define c_Ebool_2E to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2E_7E))$.

Definition 10 We define $c_Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 11 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_Emin_2E_40$

Let $c_2Eintegral_2Eintegral : \iota$ be given. Assume the following.

$$c_2Eintegral_2Eintegral \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)}})(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal) \quad (6)$$

Let $c_2Eintegral_2Eintegrable : \iota$ be given. Assume the following.

$$c_2Eintegral_2Eintegrable \in ((2^{(ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)}})(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal) \quad (7)$$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (8)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (10)$$

Definition 12 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$.

Definition 13 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 14 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2E_2C$

Let $c_2Etransc_2EDint : \iota$ be given. Assume the following.

$$c_2Etransc_2EDint \in (((2^{ty_2Erealax_2Ereal})(ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}))(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal)) \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \quad (14)$$

Assume the following.

$$(\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. (\forall V2f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). ((p\ (ap\ (ap\ c_2Eintegral_2Eintegrable\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0a)\ V1b))\ V2f)) \Leftrightarrow (\exists V3i \in ty_2Erealax_2Ereal. (p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0a)\ V1b))\ V2f)\ V3i))))))) \quad (15)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1a \in ty_2Erealax_2Ereal. (\forall V2b \in ty_2Erealax_2Ereal. ((p\ (ap\ (ap\ c_2Eintegral_2Eintegrable\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V1a)\ V2b))\ V0f)) \Rightarrow (p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V1a)\ V2b))\ V0f)\ (ap\ (ap\ c_2Eintegral_2Eintegral\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V1a)\ V2b))\ V0f))))))) \quad (16)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V2a \in ty_2Erealax_2Ereal. (\forall V3b \in ty_2Erealax_2Ereal. (\forall V4i \in ty_2Erealax_2Ereal. (\forall V5j \in ty_2Erealax_2Ereal. (((p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b))\ V0f)\ V4i)) \wedge (p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b))\ V1g)\ V5j))) \Rightarrow (p\ (ap\ (ap\ (ap\ c_2Etransc_2EDint\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b))\ (\lambda V6x \in ty_2Erealax_2Ereal. (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ V0f\ V6x))\ (ap\ V1g\ V6x))))\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V4i)\ V5j)))))))))) \quad (17)$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2a \in ty_2Erealax_2Ereal. \\
& (\forall V3b \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte \\
& V2a) V3b)) \wedge ((p (ap (ap c_2Eintegral_2Eintegrable (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V2a) V3b)) V0f)) \wedge (p (\\
& ap (ap c_2Eintegral_2Eintegrable (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V2a) V3b)) V1g)))) \Rightarrow (p (ap (ap c_2Eintegral_2Eintegrable \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V2a) V3b)) (\lambda V4x \in ty_2Erealax_2Ereal.(ap (ap c_2Erealax_2Ereal_add \\
& (ap V0f V4x)) (ap V1g V4x))))))))))
\end{aligned}$$