

thm\_2Integral\_2INTEGRABLE\_\_CAUCHY  
 (TMcTQjhDrTpWc-  
 CzC1ew4qCeSwMFVBxW5HZ6)

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Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Eintegral\_2Eintegrable : \iota$  be given. Assume the following.

$$c\_2Eintegral\_2Eintegrable \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})}) (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal)) \tag{3}$$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{4}$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Eenum\_2Eenum}) \tag{5}$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Eenum\_2Eenum}) \tag{6}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E2T$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

**Definition 4** We define  $c\_2Ebool\_2E2F$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (9)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (ap$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 12** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 14** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

**Definition 15** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2ECOND$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 16** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (14)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (15)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ t$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (16)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (17)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (18)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 19** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (19)$$

**Definition 20** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (20)$$

**Definition 21** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal} \quad (21)$$

**Definition 22** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in$



**Definition 35** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c)^{A\_27a})$ .  
Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (28)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))}) \quad (29)$$

**Definition 36** We define  $c\_2Emetric\_2Emr1$  to be  $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)))$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (30)$$

**Definition 37** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ x\ y)$ .

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))^{(2^{A\_27a})^{A\_27a}}) \quad (31)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (32)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})^{A\_27a}})}) \quad (33)$$

**Definition 38** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a). (ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27a)\ m)$ .

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a)^{A\_27b}}) \quad (34)$$

**Definition 39** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V1x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27a)\ x\ y)$ .

**Definition 40** We define  $c\_2Eseq\_2Econvergent$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27a)\ f)$ .

**Definition 41** We define  $c\_2Eseq\_2Ecauchy$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27a)\ f)$ .

Let  $c\_2Etransc\_2Ersum : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Ersum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}})})})^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}})})})})^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}})})})} \quad (35)$$

Let  $c\_2Etransc\_2EDint : \iota$  be given. Assume the following.

$$c\_2Etransc\_2EDint \in (((2^{(ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}})})})^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}})})})})^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}})})})} \quad (36)$$

Let  $c\_2Etransc\_2Efine : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Efine \in ((2^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}))})^{(ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum})})^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}))})} \quad (37)$$

Let  $c\_2Etransc\_2Etdiv : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Etdiv \in ((2^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}))})^{(ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum})})^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}))})} \quad (38)$$

**Definition 42** We define  $c\_2Etransc\_2Egauge$  to be  $\lambda V0E \in (2^{ty\_2Erealax\_2Ereal}). \lambda V1g \in (ty\_2Erealax\_2Ereal$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V0m)))))) \quad (41)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = c\_2Enum\_2E0) \wedge (V1n = c\_2Enum\_2E0)))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V0m))) \quad (43)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3E\_3D V0n) V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V0n)))) \quad (44)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \wedge (\forall V1m \in ty\_2Enum\_2Enum. \\
& (\forall V2n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) (ap c\_2Enum\_2ESUC V2n))) \Leftrightarrow ((V1m = (ap c\_2Enum\_2ESUC V2n)) \vee \\
& (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V2n)))))))
\end{aligned} \tag{45}$$

Assume the following.

$$True \tag{46}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{47}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\
A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{50}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (55)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (56)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (57)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (59)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))))) \quad (60)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p (ap V0P V2x)))))) \quad (61)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a.(p (ap V1Q V4x)))))) \quad (62)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p V0P) \wedge (\forall V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (63)$$



Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((p\ V0P) \vee (\exists V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow (\exists V3x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \quad (64)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (65)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in 2. ((\forall V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a. (p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (69)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (70)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (71)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))))) \Rightarrow ((\exists V3x \in A\_27a. (p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a. (p\ (ap\ V1Q\ V4x)))))) \quad (72)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\
& \quad \forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in \\
& A\_27b.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).( \\
& \quad \forall V4x \in A\_27a.(p (ap (ap V0P V4x) (ap V3f V4x))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in \text{ty\_2Erealax\_2Ereal}.\forall V1b \in \text{ty\_2Erealax\_2Ereal}. \\
& \quad (\forall V2f \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}}).((p ( \\
& ap (ap c\_2Eintegral\_2Eintegrable (ap (ap (c\_2Epair\_2E\_2C \text{ty\_2Erealax\_2Ereal} \\
& \quad \text{ty\_2Erealax\_2Ereal}) V0a) V1b)) V2f)) \Leftrightarrow (\exists V3i \in \text{ty\_2Erealax\_2Ereal}. \\
& (p (ap (ap (ap c\_2Etransc\_2EDint (ap (ap (c\_2Epair\_2E\_2C \text{ty\_2Erealax\_2Ereal} \\
& \quad \text{ty\_2Erealax\_2Ereal}) V0a) V1b)) V2f) V3i))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in \text{ty\_2Erealax\_2Ereal}.\forall V1b \in \text{ty\_2Erealax\_2Ereal}. \\
& \quad (\forall V2f \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}}).(\forall V3i \in \\
& \quad \text{ty\_2Erealax\_2Ereal}.\text{lt } V1b) V0a)) \Rightarrow \\
& (p (ap (ap (ap c\_2Etransc\_2EDint (ap (ap (c\_2Epair\_2E\_2C \text{ty\_2Erealax\_2Ereal} \\
& \quad \text{ty\_2Erealax\_2Ereal}) V0a) V1b)) V2f) V3i))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in \text{ty\_2Erealax\_2Ereal}.\exists V1n \in \text{ty\_2Enum\_2Enum}. \\
& (p (ap (ap (ap c\_2Erealax\_2Ereal\_lt V0x) (ap (ap c\_2Ereal\_2Epow (ap \\
& c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
& \quad c\_2Earithmetic\_2EZERO)))) V1n))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{\text{ty\_2Erealax\_2Ereal}}).\forall V1gs \in ((\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}})^{\text{ty\_2Enum\_2Enum}}). \\
& \quad (\forall V2n \in \text{ty\_2Enum\_2Enum}.\text{lt } V2n). \\
& \quad ((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D V3m) V2n)) \Rightarrow (p (ap (ap c\_2Etransc\_2Egauge \\
& \quad V0s) (ap V1gs V3m)))) \Rightarrow (\exists V4g \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}}). \\
& (p (ap (ap c\_2Etransc\_2Egauge V0s) V4g)) \wedge (\forall V5d \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Enum\_2Enum}}). \\
& \quad (\forall V6p \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Enum\_2Enum}}).\text{lt } V6p)) \Rightarrow (\forall V7m \in \\
& \quad \text{ty\_2Enum\_2Enum}.\text{lt } V7m) \Rightarrow \\
& \quad (p (ap (ap c\_2Etransc\_2Efine (ap V1gs V7m)) (ap (ap (c\_2Epair\_2E\_2C \\
& \quad (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Enum\_2Enum}}) V5d) V6p)))))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) (ap c\_2Earithmic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\
& (ap c\_2Earithmic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n))) \wedge ((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))))))))))))) \\
& \tag{79}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& V1y) V0x)))) \\
& \tag{80}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) (ap (ap c\_2Erealax\_2Ereal\_add V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z)))))) \\
& \tag{81}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = V0x)) \\
& \tag{82}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \\
& \tag{83}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt V0x) V2z)))))) \\
& \tag{84}
\end{aligned}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V0x)))) \quad (85)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x)) \quad (86)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_add V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x)) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_add V0x) (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (88)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) = (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Erealax\_2Ereal\_neg V0x)) (ap c\_2Erealax\_2Ereal\_neg V1y)))))) \quad (89)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (90)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V2z))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z)))))) \quad (91)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \vee (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V0x)))))) \quad (92)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V0x))) \quad (93)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))) \quad (94)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))) \quad (95)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V2z))))))) \quad (96)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V0x))) \Leftrightarrow (V0x = V1y)))) \quad (97)$$

Assume the following.

$$(p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \quad (98)$$

Assume the following.

$$(\forall V0w \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal. (\forall V2y \in ty\_2Erealax\_2Ereal.(\forall V3z \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Erealax\_2Ereal\_lt V0w) V1x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V2y) V3z))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap c\_2Erealax\_2Ereal\_add V0w) V2y)) (ap (ap c\_2Erealax\_2Ereal\_add V1x) V3z)))))))))) \quad (99)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add (ap (ap c\_2Ereal\_2Ereal\_sub V0x) V1y)) V1y) = V0x))) \quad (100)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \Rightarrow \\
& (V0x = (ap c\_2Erealax\_2Einv V1y))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Einv V0x) = \\
& (ap (ap c\_2Ereal\_2E\_2F (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x)))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0m) V1n))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((\neg (V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \Rightarrow ((ap (ap c\_2Ereal\_2E\_2F V0x) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Ereal\_2E\_2F V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) (ap (ap \\
& c\_2Ereal\_2E\_2F V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) = V0x))
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y))) \Rightarrow \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Erealax\_2Einv V1y)) (ap \\
& c\_2Erealax\_2Einv V0x))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V1y))) \Rightarrow (p (ap (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2E\_2F V0x) V1y))))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (((p (ap (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V2z))) \Rightarrow (p (ap (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V0x)) (ap (ap c\_2Erealax\_2Ereal\_mul V2z) V0x))))))
\end{aligned} \tag{109}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((V0x = V1y) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y))))
\end{aligned} \tag{110}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((p (ap (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) \Rightarrow (\neg (V0x = (ap \\
& c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{111}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Ereal\_2Eabs (ap c\_2Erealax\_2Ereal\_neg \\
& V0x)) = (ap c\_2Ereal\_2Eabs V0x)))
\end{aligned} \tag{112}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (p (ap (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y))) (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Ereal\_2Eabs \\
& V0x)) (ap c\_2Ereal\_2Eabs V1y))))))
\end{aligned} \tag{113}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (ap (ap c\_2Ereal\_2Epow \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) V0n))))
\end{aligned} \tag{114}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0x)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2Epow V0x) V1n))))))
\end{aligned} \tag{115}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) V1y))))))
\end{aligned} \tag{116}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0y) V1x)))))
\end{aligned} \tag{117}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V2z)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V2z))))))
\end{aligned} \tag{118}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y))))))
\end{aligned} \tag{119}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x))))))
\end{aligned} \tag{120}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x))
\end{aligned} \tag{121}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Erealax\_2Ereal\_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{122}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1m \in ty\_2Enum\_2Enum. \\
& (\forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ereal\_2Epow V0x) (ap \\
& (ap c\_2Earithmetic\_2E\_2B V1m) V2n)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Ereal\_2Epow V0x) V1m)) (ap (ap c\_2Ereal\_2Epow V0x) V2n))))))
\end{aligned} \tag{123}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0) V2z)) \Rightarrow ((p (ap (ap \\
& c\_2Erealax\_2Ereal\_lt (ap (ap c\_2Ereal\_2E\_2F V0x) V2z)) V1y)) \Leftrightarrow \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z))))))
\end{aligned} \tag{124}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z))))))
\end{aligned} \tag{125}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n))))
\end{aligned} \tag{126}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Ereal\_2Eabs \\
& V0x)) (ap c\_2Ereal\_2Eabs V1y))))))
\end{aligned} \tag{127}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1z \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Ereal\_2E\_2F V0y) (ap \\
& c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
& c\_2Earithmetic\_2EZERO)))))) (ap (ap c\_2Ereal\_2E\_2F (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) \\
& V1z)) = (ap (ap c\_2Ereal\_2E\_2F V0y) V1z)))
\end{aligned} \tag{128}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{129}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{130}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{131}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{132}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{133}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\
& p V2r)) \vee (\neg(p V1q)))))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{134}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{135}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r)))))) \wedge \\
& (((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{136}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (137)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (138)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (139)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q)))) \quad (140)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p)))) \quad (141)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (142)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (143)$$

Assume the following.

$$(\forall V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1x0 \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Eseq\_2E\_2D\_2D\_3E V0x) V1x0)) \Leftrightarrow (\forall V2e \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V2e)) \Rightarrow (\exists V3N \in ty\_2Enum\_2Enum. (\forall V4n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3E\_3D V4n) V3N)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub (ap V0x V4n)) V1x0)) V2e)))))))))) \quad (144)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). ((p (ap c\_2Eseq\_2Ecauchy V0f)) \Leftrightarrow (p (ap c\_2Eseq\_2Econvergent V0f)))) \quad (145)$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1b \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V3k \in \\
& \quad ty\_2Erealax\_2Ereal. ((p (ap (ap (ap c\_2Etransc\_2EDint (ap (ap ( \\
& \quad c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0a) \\
& \quad V1b)) V2f) V3k))) \Leftrightarrow (\forall V4e \in ty\_2Erealax\_2Ereal. ((p (ap (ap \\
& \quad c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& \quad V4e))) \Rightarrow (\exists V5g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap c\_2Etransc\_2Egauge (\lambda V6x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Ereal\_2Ereal\_lte V0a) V6x)) \\
& \quad (ap (ap c\_2Ereal\_2Ereal\_lte V6x) V1b)))) V5g)) \wedge (\forall V7D \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V8p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& \quad (((p (ap (ap c\_2Etransc\_2Etdiv (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V0a) V1b)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V7D) V8p))) \wedge (p (ap (ap c\_2Etransc\_2Efine \\
& \quad V5g) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V7D) V8p)))) \Rightarrow (p (ap (ap \\
& \quad c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub \\
& \quad (ap (ap c\_2Etransc\_2Ersum (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V7D) V8p)) V2f)) V3k))) \\
& \quad V4e))))))))))))) \\
& \hspace{15em} (146)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1b \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V2g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (((p \\
& \quad (ap (ap c\_2Ereal\_2Ereal\_lte V0a) V1b)) \wedge (p (ap (ap c\_2Etransc\_2Egauge \\
& \quad (\lambda V3x \in ty\_2Erealax\_2Ereal. (ap (ap c\_2Ebool\_2E\_2F\_5C (ap ( \\
& \quad ap c\_2Ereal\_2Ereal\_lte V0a) V3x)) (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V3x) V1b)))) V2g))) \Rightarrow (\exists V4D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& \quad (\exists V5p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). ((p (ap (ap \\
& \quad c\_2Etransc\_2Etdiv (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V0a) V1b)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V4D) V5p))) \wedge (p (ap (ap c\_2Etransc\_2Efine \\
& \quad V2g) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V4D) V5p))))))))) \\
& \hspace{15em} (147)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in \\
& ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap c\_2Eintegral\_2Eintegrable (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V1a) V2b)) V0f))) \Leftrightarrow (\forall V3e \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V3e))) \Rightarrow (\exists V4g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap c\_2Etrasc\_2Egauge (\lambda V5x \in ty\_2Erealax\_2Ereal. \\
& (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Ereal\_2Ereal\_lte V1a) V5x)) \\
& (ap (ap c\_2Ereal\_2Ereal\_lte V5x) V2b)))) V4g)) \wedge (\forall V6d1 \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V7p1 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& (\forall V8d2 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V9p2 \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(((p (ap (ap c\_2Etrasc\_2Etdiv \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V1a) V2b)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V6d1) V7p1)))) \wedge ((p (ap ( \\
& ap c\_2Etrasc\_2Efine V4g) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V6d1) V7p1)))) \wedge ((p (ap ( \\
& ap c\_2Etrasc\_2Etdiv (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V1a) V2b)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V8d2) V9p2)))) \wedge (p (ap (ap \\
& c\_2Etrasc\_2Efine V4g) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V8d2) V9p2)))))) \Rightarrow (p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub \\
& (ap (ap c\_2Etrasc\_2Ersum (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V6d1) V7p1)) V0f)) (ap ( \\
& ap c\_2Etrasc\_2Ersum (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V8d2) V9p2)) V0f)))) V3e)))))))))
\end{aligned}$$