

thm\_2Eintegral\_2EINTEGRAL\_\_BY\_\_PARTS  
(TMTPLk-  
WgZ91j5VcKHB4UjJWTvzNc28evt1q)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Etransc\_2EDint : \iota$  be given. Assume the following.

$$c\_2Etransc\_2EDint \in (((2^{ty\_2Erealax\_2Ereal})^{(ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal}})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{3}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{4}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{6}$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (9)$$

**Definition 4** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 5** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E3D (2^{A-27a}$

**Definition 6** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 (ty$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (10)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (12)$$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

**Definition 8** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 10** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

**Definition 11** We define `c_2Erealax_2Einv` to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS)$   
Let `c_2Erealax_2Etreal_mul` :  $\iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)} \quad (15)$$

**Definition 12** We define `c_2Erealax_2Ereal_mul` to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 13** We define `c_2Ereal_2E_2F` to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$   
Let `c_2Erealax_2Etreal_lt` :  $\iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)} \quad (16)$$

**Definition 14** We define `c_2Erealax_2Ereal_lt` to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 15** We define `c_2Ebool_2EF` to be  $(ap\ (c\_2Ebool\_2E\_21\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 16** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$   
of type  $\iota$ .

**Definition 17** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t))\ c\_2Ebool\_2E\_7E)$

**Definition 18** We define `c_2Ereal_2Ereal_lte` to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 19** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2))\ (\lambda V2t \in 2.V2t)))$

**Definition 20** We define `c_2Ebool_2ECOND` to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.$

**Definition 21** We define `c_2Ereal_2Eabs` to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

Let `c_2Epair_2ESND` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (17)$$

Let `c_2Epair_2EFST` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (18)$$

**Definition 22** We define `c_2Epair_2EUNCURRY` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27b})$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (19)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (20)$$

**Definition 23** We define  $c\_2Emetric\_2Emr1$  to be  $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epair\_2EABS\_prod\ : \iota \Rightarrow \iota \Rightarrow \iota)$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (21)$$

**Definition 24** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ : \iota \Rightarrow \iota \Rightarrow \iota)$

Let  $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2Emetric\ A\_27a)\ A\_27a)}) \quad (22)$$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(c\_2Emetric\_2Emetric\ A\_27a)}) \quad (23)$$

**Definition 25** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ : \iota \Rightarrow \iota \Rightarrow \iota)$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (24)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (25)$$

**Definition 26** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap\ (c\_2Emin\_2E\_40\ : \iota \Rightarrow \iota \Rightarrow \iota)$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A\_27b})^{A\_27b})})^{A\_27a})^{(A\_27a)^{A\_27b}}) \quad (26)$$

**Definition 27** We define  $c\_2Elim\_2Etends\_real\_real$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$

**Definition 28** We define  $c\_2Elim\_2Ediff1$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) . \lambda V1l \in ty\_2$

Let  $c\_2Eintegral\_2Eintegral : \iota$  be given. Assume the following.

$$c\_2Eintegral\_2Eintegral \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) (27)$$

Let  $c\_2Eintegral\_2Eintegrable : \iota$  be given. Assume the following.

$$c\_2Eintegral\_2Eintegrable \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) (28)$$

**Definition 29** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) (29)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) (30)$$

**Definition 30** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum . (ap\ c\_2Enum\_2EABS\_num)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) (31)$$

**Definition 31** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum . (ap\ (ap\ c\_2Earithmetic\_2E\_2B))$

**Definition 32** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum . V0x$ .

**Definition 33** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2 . (\lambda V1t2 \in 2 . (ap\ (c\_2Ebool\_2E\_21\ 2))\ (\lambda V2t \in 2 . (ap\ (c\_2Ebool\_2E\_21\ 2))\ t))))$

Assume the following.

$$True (32)$$

Assume the following.

$$\forall A\_27a . nonempty\ A\_27a \Rightarrow (\forall V0t \in 2 . ((\forall V1x \in A\_27a . (p\ V0t)) \Leftrightarrow (p\ V0t))) (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2 . (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) (34) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2 . (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) (35) \end{aligned}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (41)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow ((\forall V3x \in A.27a.(p (ap V0P V3x))) \Rightarrow (\forall V4x \in A.27a.(p (ap V1Q V4x)))))) \quad (42)$$

Assume the following.

$$(\forall V0f \in (ty.2Erealx.2Ereal^{ty.2Erealx.2Ereal}).(\forall V1a \in ty.2Erealx.2Ereal.(\forall V2b \in ty.2Erealx.2Ereal.(\forall V3i \in ty.2Erealx.2Ereal.(((p (ap (ap c.2Ereal.2Ereal\_lte V1a) V2b)) \wedge (p (ap (ap (ap c.2Etransc.2EDint (ap (ap (c.2Epair.2E.2C ty.2Erealx.2Ereal ty.2Erealx.2Ereal) V1a) V2b)) V0f) V3i))) \Rightarrow ((ap (ap c.2Eintegral.2Eintegral (ap (ap (c.2Epair.2E.2C ty.2Erealx.2Ereal ty.2Erealx.2Ereal) V1a) V2b)) V0f) = V3i)))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2f\_27 \in ( \\
ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V3g\_27 \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& \quad (\forall V4a \in ty\_2Erealax\_2Ereal.(\forall V5b \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Ereal\_2Ereal\_lte V4a) V5b)) \wedge ((\forall V6x \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Ereal\_2Ereal\_lte V4a) V6x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V6x) V5b))) \Rightarrow (p (ap (ap (ap c\_2Elim\_2Ediff V0f) (ap V2f\_27 V6x)) \\
& \quad V6x)))) \wedge (\forall V7x \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V4a) V7x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V7x) V5b))) \Rightarrow (p (ap ( \\
& \quad ap (ap c\_2Elim\_2Ediff V1g) (ap V3g\_27 V7x)) V7x)))))) \Rightarrow (p (ap (ap \\
& \quad (ap c\_2Etransc\_2EDint (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V4a) V5b)) (\lambda V8x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_add (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad (ap V2f\_27 V8x)) (ap V1g V8x))) (ap (ap c\_2Erealax\_2Ereal\_mul ( \\
& \quad ap V0f V8x)) (ap V3g\_27 V8x)))))) (ap (ap c\_2Ereal\_2Ereal\_sub (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_mul (ap V0f V5b)) (ap V1g V5b))) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad (ap V0f V4a)) (ap V1g V4a)))))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2a \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V3b \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V2a) V3b)) \wedge ((p (ap (ap c\_2Eintegral\_2Eintegrable (ap (ap (c\_2Epair\_2E\_2C \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V2a) V3b)) V0f)) \wedge (p ( \\
& \quad ap (ap c\_2Eintegral\_2Eintegrable (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V2a) V3b)) V1g)))) \Rightarrow ((ap (ap c\_2Eintegral\_2Eintegral \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad V2a) V3b)) (\lambda V4x \in ty\_2Erealax\_2Ereal.(ap (ap c\_2Erealax\_2Ereal\_add \\
& \quad (ap V0f V4x)) (ap V1g V4x)))) = (ap (ap c\_2Erealax\_2Ereal\_add (ap \\
& \quad (ap c\_2Eintegral\_2Eintegral (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V2a) V3b)) V0f)) (ap (ap c\_2Eintegral\_2Eintegral \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad V2a) V3b)) V1g)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad ((ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& \quad V1y) V0x))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
V0x) (ap (ap c\_2Erealax\_2Ereal\_add V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
(ap c\_2Erealax\_2Ereal\_neg V0x)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
(ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
V0x) (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
((ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_add V0x) \\
& V1y)) = (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Erealax\_2Ereal\_neg \\
V0x)) (ap c\_2Erealax\_2Ereal\_neg V1y))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
(ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
V0x) V0x)))
\end{aligned} \tag{54}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x)))) \Leftrightarrow (V0x = V1y)))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) V1y))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0y) V1x)))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x)))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Erealax\_2Ereal\_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z)))))
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (69)$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& \quad (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2f\_27 \in ( \\
ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V3g\_27 \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& \quad (\forall V4a \in ty\_2Erealax\_2Ereal.(\forall V5b \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Ereal\_2Ereal\_lte V4a) V5b)) \wedge ((\forall V6x \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Ereal\_2Ereal\_lte V4a) V6x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V6x) V5b)))) \Rightarrow (p (ap (ap (ap c\_2Elim\_2Ediff V0f) (ap V2f\_27 V6x)) \\
& \quad V6x)))) \wedge ((\forall V7x \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V4a) V7x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V7x) V5b)))) \Rightarrow (p (ap ( \\
& \quad ap (ap c\_2Elim\_2Ediff V1g) (ap V3g\_27 V7x)) V7x)))) \wedge (p (ap (ap \\
& \quad c\_2Eintegral\_2Eintegrable (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V4a) V5b)) (\lambda V8x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_mul (ap V2f\_27 V8x)) (ap V1g V8x)))))) \wedge \\
& \quad (p (ap (ap c\_2Eintegral\_2Eintegrable (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V4a) V5b)) (\lambda V9x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_mul (ap V0f V9x)) (ap V3g\_27 V9x)))))) \Rightarrow \\
& \quad ((ap (ap c\_2Eintegral\_2Eintegral (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V4a) V5b)) (\lambda V10x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_mul (ap V0f V10x)) (ap V3g\_27 V10x)))) = \\
& \quad (ap (ap c\_2Ereal\_2Ereal\_sub (ap (ap c\_2Ereal\_2Ereal\_sub (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_mul (ap V0f V5b)) (ap V1g V5b))) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad (ap V0f V4a)) (ap V1g V4a)))) (ap (ap c\_2Eintegral\_2Eintegral (ap \\
& \quad (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal \\
& \quad V4a) V5b)) (\lambda V11x \in ty\_2Erealax\_2Ereal.(ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad (ap V2f\_27 V11x)) (ap V1g V11x)))))))))
\end{aligned}$$