

thm_2Eintegral_2EINTEGRAL__CMUL
(TMK54Wcjs7KzeD54K9XgvsA7s88HhrECCR5)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2E$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Eintegral_2Eintegrable : \iota$ be given. Assume the following.

$$c_2Eintegral_2Eintegrable \in ((2^{(ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)}) \tag{3}$$

Let $c_2Eintegral_2Eintegral : \iota$ be given. Assume the following.

$$c_2Eintegral_2Eintegral \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)}) \tag{4}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{5}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{6}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 8 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$
Let $c_2Erealax_2Ereal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)} \quad (7)$$

Definition 9 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 10 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$
Let $c_2Erealax_2Ereal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)} \quad (8)$$

Let $c_2Erealax_2Ereal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)} \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}} \quad (10)$$

Definition 12 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 13 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Etransc_2EDint : \iota$ be given. Assume the following.

$$c_2Etransc_2EDint \in (((2^{ty_2Erealax_2Ereal})^{(ty_2Erealax_2Ereal)^{ty_2Erealax_2Ereal}})^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal)})^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal)} \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1a \in \\
& ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.((p \ (ap \\
& (ap \ c_2Eintegral_2Eintegrable \ (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) \ V1a) \ V2b)) \ V0f)) \Rightarrow (p \ (ap \ (ap \ (ap \ c_2Etransc_2EDint \\
& (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \\
& V1a) \ V2b)) \ V0f) \ (ap \ (ap \ c_2Eintegral_2Eintegral \ (ap \ (ap \ (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \ V1a) \ V2b)) \ V0f))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1a \in \\
& ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.(\forall V3i \in \\
& ty_2Erealax_2Ereal.(((p \ (ap \ (ap \ c_2Ereal_2Ereal_lte \ V1a) \ V2b)) \wedge \\
& (p \ (ap \ (ap \ (ap \ c_2Etransc_2EDint \ (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) \ V1a) \ V2b)) \ V0f) \ V3i))) \Rightarrow ((ap \ (ap \ c_2Eintegral_2Eintegral \\
& (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \\
& V1a) \ V2b)) \ V0f) = V3i))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1a \in \\
& ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.(\forall V3c \in \\
& ty_2Erealax_2Ereal.(\forall V4i \in ty_2Erealax_2Ereal.((p \ (ap \\
& (ap \ (ap \ c_2Etransc_2EDint \ (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) \ V1a) \ V2b)) \ V0f) \ V4i)) \Rightarrow (p \ (ap \ (ap \ (ap \ c_2Etransc_2EDint \\
& (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \\
& V1a) \ V2b)) \ (\lambda V5x \in ty_2Erealax_2Ereal.(ap \ (ap \ c_2Erealax_2Ereal_mul \\
& V3c) \ (ap \ V0f \ V5x))) \ (ap \ (ap \ c_2Erealax_2Ereal_mul \ V3c) \ V4i))))))
\end{aligned} \tag{18}$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1c \in \\ & ty_2Erealax_2Ereal.(\forall V2a \in ty_2Erealax_2Ereal.(\forall V3b \in \\ & ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V2a) V3b)) \wedge \\ (p (ap (ap c_2Eintegral_2Eintegrable (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ ty_2Erealax_2Ereal) V2a) V3b)) V0f))) \Rightarrow ((ap (ap c_2Eintegral_2Eintegral \\ (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ V2a) V3b)) (\lambda V4x \in ty_2Erealax_2Ereal.(ap (ap c_2Erealax_2Ereal_mul \\ V1c) (ap V0f V4x)))) = (ap (ap c_2Erealax_2Ereal_mul V1c) (ap (ap \\ c_2Eintegral_2Eintegral (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ ty_2Erealax_2Ereal) V2a) V3b)) V0f)))))))))) \end{aligned}$$