

# thm\_2Eintegral\_2ETDIV\_\_LE (TMJURKN- jmu1HjBfQ9YLRJV3PJmkiroqKNwJ)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

**Definition 5** We define `c_2Ecombin_2EK` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V0x \in A. 27a. (\lambda V1y \in A. 27b. V0x))$

**Definition 6** We define `c_2Ecombin_2ES` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A. 27c^{A-27b})^{A-27a}))$

**Definition 7** We define `c_2Ecombin_2EI` to be  $\lambda A. 27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a (A. 27a^{A-27a})) A_27a))$

**Definition 8** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a}) P))))$

**Definition 9** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 10** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2. V2t)))$

**Definition 12** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_5C_2F))$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let `c_2Enum_2EREP__num` :  $\iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let `c_2Enum_2ESUC__REP` :  $\iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let `c_2Enum_2EABS__num` :  $\iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 13** We define  $c\_Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 14** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (5)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Etransc\_2Etdiv : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Etdiv \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}})) \quad (7)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}))_{ty\_2Erealax} \quad (9)$$

**Definition 15** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ (t$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}))_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal)} \quad (10)$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 17** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (11)$$

**Definition 18** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let  $c\_2Etransc\_2Edivision : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Edivision \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})_{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal)})) \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))) \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False) \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p (ap V0P V2x)))))) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ ((\forall V3x \in A.27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p ( \\ ap V1Q V4x))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\ A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ (2^{A.27a}).((\exists V2x \in A.27a.((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow \\ ((\exists V3x \in A.27a.(p (ap V0P V3x))) \vee (\exists V4x \in A.27a.(p ( \\ ap V1Q V4x)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in \\ A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x)))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee \\ (p V0A)))) \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).( \\ \forall V4x \in A.27a.(p (ap (ap V0P V4x) (ap V3f V4x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c.2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1b \in ty\_2Erealax\_2Ereal. \\ & (\forall V2D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V3p \in \\ & (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). ((p (ap (ap c\_2Etransc\_2Etdiv \\ & (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ V0a) V1b)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\ & (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V2D) V3p)))) \Leftrightarrow ((p (ap (ap \\ & c\_2Etransc\_2Edivision (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) V0a) V1b)) V2D)) \wedge (\forall V4n \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap V2D V4n)) (ap V3p V4n))) \wedge (p \\ & (ap (ap c\_2Ereal\_2Ereal\_lte (ap V3p V4n)) (ap V2D (ap c\_2Enum\_2ESUC \\ & V4n))))))))))))) \quad (43) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1a \in \\ & ty\_2Erealax\_2Ereal. (\forall V2b \in ty\_2Erealax\_2Ereal. ((p (ap \\ & (ap c\_2Etransc\_2Edivision (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) V1a) V2b)) V0D)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\ & V1a) V2b)))))) \quad (44) \end{aligned}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0d \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1p \in \\ & (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V2a \in ty\_2Erealax\_2Ereal. \\ & (\forall V3b \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Etransc\_2Etdiv \\ & (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & V2a) V3b)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \\ & (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})) V0d) V1p)))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\ & V2a) V3b)))))) \end{aligned}$$