

thm_2Eintreal_2Ereal_of_int_11
 (TMZ8XUjJ4V8FVLL8qCZ2U8LNVto5mDngAp6)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p x)) \text{ else } (\lambda x.x \in A \wedge \neg p x)$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))))$

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Epair_2Eprod \\ A0 A1) \end{aligned} \quad (2)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Einteger_2Eint \quad (3)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint} \quad (4)$$

Definition 9 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ (ty_2Einteger_2Eint\ a)\ b)\ c)$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (5)$$

Let $c \in \mathbb{Z}$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum)} \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})})$$

Definition 10 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Eint)$

Definition 11 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (8)$$

Definition 12 We define $c_2Einteger_2EEnum$ to be $\lambda V o \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E40\ ty_2Eint)\ o)$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (9)$$

Definition 13 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger.$

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A.\lambda 27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Let $ty_2Ereal _2Ereal : \iota$ be given. Assume the following.

nonempty *ty_2Erealax_2Ereal* (10)

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

nonempty t_2 \in $Ehreal$ $\setminus Ehreal$

Let $c : 2\mathit{Frealax} \cdot 2\mathit{Freal} \cdot \mathit{REP} \cdot \mathit{CLASS} : t$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})ty_2Erealax) \\ (13)$$

Definition 16 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$
Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} \quad (16)$$

Definition 17 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 18 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap (c_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Let $c_2Enum_2EAbs_num : \iota$ be given. Assume the following.

$$c_2Enum_2EAbs_num \in (ty_2Enum_2Enum^{\omega}) \quad (18)$$

Definition 19 We define c_2Enum_2E0 to be $(ap (c_2Enum_2EAbs_num c_2Enum_2EZERO_REP))$.

Definition 20 We define $c_2Eintreal_2Ereal_of_int$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap (ap (ap (c_2Ebo$

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap (c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (20)$$

Definition 22 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap (c_2Enum_2EAbs_num$

Definition 23 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ True)) \quad (27)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \\ & V0t))))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a. & nonempty A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))) \\ (32) \end{aligned}$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. ((ap c_2Einteger_2Eint_neg \\ (ap c_2Einteger_2Eint_neg V0x)) = V0x)) \quad (33)$$

Assume the following.

$$((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ ((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\ V1n)) \Leftrightarrow (V0m = V1n))) \\ (35) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\ V0n)) (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\ V0n) V1m))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\ (ap c_2Einteger_2Eint_of_num V0n))) (ap c_2Einteger_2Eint_neg \\ (ap c_2Einteger_2Eint_of_num V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\ V1m) V0n))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\ (ap c_2Einteger_2Eint_of_num V0n))) (ap c_2Einteger_2Eint_of_num \\ V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0))) \wedge ((p \\ (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\ V0n)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ V1m)))) \Leftrightarrow False)))))) \\ (36) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0p \in ty_2Einteger_2Eint. ((\exists V1n \in ty_2Enum_2Enum. \\ ((V0p = (ap c_2Einteger_2Eint_of_num V1n)) \wedge (\neg(V1n = c_2Enum_2E0)))) \vee \\ ((\exists V2n \in ty_2Enum_2Enum. ((V0p = (ap c_2Einteger_2Eint_neg \\ (ap c_2Einteger_2Eint_of_num V2n))) \wedge (\neg(V2n = c_2Enum_2E0)))) \vee \\ (V0p = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))))) \\ (37) \end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Einteger_2ENum (ap c_2Einteger_2Eint_of_num \\ V0n)) = V0n)) \quad (38)$$

Assume the following.

$((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num V1n)) \Leftrightarrow (V0m = V1n)))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. (\forall V3y \in ty_2Einteger_2Eint. (((ap c_2Einteger_2Eint_neg V2x) = (ap c_2Einteger_2Eint_neg V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty_2Enum_2Enum. (((ap c_2Einteger_2Eint_of_num V4n) = (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V5m))) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V4n)) = (ap c_2Einteger_2Eint_of_num V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))))))$
(39)

Assume the following.

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \quad (41)$$

Assume the following.

$$((ap\ c_2Ereal_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = \\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \quad (42)$$

Assume the following.

Theorem 1

$$((\forall V0m \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Einteger_2Eint. ((ap c_2Eintreal_2Ereal_of_int V0m) = (ap c_2Eintreal_2Ereal_of_int V1n)) \Leftrightarrow (V0m = V1n))))$$