

thm\_2Eintreal\_2Ereal\_of\_int\_neg  
(TMHXLZE9UU3ipaW3kxpCA439ACQYgBRomGV)

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**Definition 1** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

**Definition 5** We define `c_2Ebool_2E_2F` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define `c_2Ebool_2E_3F` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A P)))$

**Definition 8** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let `ty_2Einteger_2Eint` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let `c_2Einteger_2Eint_REP_CLASS` :  $\iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})ty\_2Einteger\_2Eint) \tag{4}$$

**Definition 9** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E40 (ty$

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)) \quad (5)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)) \quad (6)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})} \quad (7)$$

**Definition 10** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)$

**Definition 11** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap c\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint)^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 12** We define  $c\_2Einteger\_2ENum$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E40 ty\_2Eint$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)) \quad (9)$$

**Definition 13** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 14** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21 2) (\lambda V2t \in$

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealax\_2Ereal \quad (10)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (11)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehreal\_2Ehreal \quad (12)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Erealax\_2Ereal)) \quad (13)$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E.40 (t$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)}) \quad (15)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})} \quad (16)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (17)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (18)$$

**Definition 19** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 20** We define  $c\_2Eintreal\_2Ereal\_of\_int$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint.(ap (ap (ap (c\_2Ebo$

**Definition 21** We define  $c\_2Ebool\_2E.7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E.3D\_3D\_3E V0t) c\_2Ebool\_2E$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega)^{ty\_2Enum\_2Enum} \quad (19)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega)^{\omega} \quad (20)$$

**Definition 22** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 23** We define  $c\_2Eprim\_rec\_2E.3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND \ A\_27a) \\ & V0P) \ V2x) \ V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND \ A\_27a) \ V1Q) \ V3x\_27) \\ & \ V5y\_27)))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. ((ap (ap (ap (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \\ & \ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\ & (ap (ap (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \ V2t1) \ V3t2) = V3t2)))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_neg\ V0x)) = V0x)) \quad (32)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_neg\ V0x))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))\ V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))\ (ap\ c\_2Einteger\_2Eint\_neg\ V0x))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ V0x)\ (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0)))) \quad (34)$$

Assume the following.

$$((ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0)) = (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0)) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ c\_2Einteger\_2Eint\_of\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_of\_num\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (36)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V1m)))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1m)\ V0n))) \wedge (((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V1m))) \Leftrightarrow ((\neg(V0n = c\_2Enum\_2E0)) \vee (\neg(V1m = c\_2Enum\_2E0)))) \wedge ((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V1m)))) \Leftrightarrow False)))))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty\_2Einteger\_2Eint. ((\exists V1n \in ty\_2Enum\_2Enum. \\
& ((V0p = (ap\ c\_2Einteger\_2Eint\_of\_num\ V1n)) \wedge (\neg(V1n = c\_2Enum\_2E0)))) \vee \\
& ((\exists V2n \in ty\_2Enum\_2Enum. ((V0p = (ap\ c\_2Einteger\_2Eint\_neg \\
& (ap\ c\_2Einteger\_2Eint\_of\_num\ V2n))) \wedge (\neg(V2n = c\_2Enum\_2E0)))) \vee \\
& (V0p = (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap\ c\_2Einteger\_2ENum\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V0n)) = V0n)) \tag{39}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (((ap\ c\_2Einteger\_2Eint\_of\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V1n)) \Leftrightarrow (V0m = V1n))) \wedge ((\forall V2x \in ty\_2Einteger\_2Eint. (\forall V3y \in \\
& ty\_2Einteger\_2Eint. (((ap\ c\_2Einteger\_2Eint\_neg\ V2x) = (ap\ c\_2Einteger\_2Eint\_neg \\
& V3y)) \Leftrightarrow (V2x = V3y))) \wedge (\forall V4n \in ty\_2Enum\_2Enum. (\forall V5m \in \\
& ty\_2Enum\_2Enum. (((ap\ c\_2Einteger\_2Eint\_of\_num\ V4n) = (ap \\
& c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num\ V5m))) \Leftrightarrow \\
& ((V4n = c\_2Enum\_2E0) \wedge (V5m = c\_2Enum\_2E0)) \wedge (((ap\ c\_2Einteger\_2Eint\_neg \\
& (ap\ c\_2Einteger\_2Eint\_of\_num\ V4n)) = (ap\ c\_2Einteger\_2Eint\_of\_num \\
& V5m)) \Leftrightarrow ((V4n = c\_2Enum\_2E0) \wedge (V5m = c\_2Enum\_2E0))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty\_2Einteger\_2Eint. ((ap\ c\_2Eintreal\_2Ereal\_of\_int \\
& V0i) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Erealax\_2Ereal)\ (ap\ (ap \\
& c\_2Einteger\_2Eint\_lt\ V0i)\ (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))) \\
& (ap\ c\_2Erealax\_2Ereal\_neg\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap \\
& c\_2Einteger\_2ENum\ (ap\ c\_2Einteger\_2Eint\_neg\ V0i))))))\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& (ap\ c\_2Einteger\_2ENum\ V0i))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ c\_2Enum\_2E0)))) \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ c\_2Erealax\_2Ereal\_neg \\
& (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)) = V0x))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& ((ap\ c\_2Erealax\_2Ereal\_neg\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) = \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& \quad (((ap\ c\_2Ereal\_2Ereal\_of\_num\ V0n) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Erealax\_2Ereal\_neg\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
V0n)) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ V1m)) \Leftrightarrow ((V0n = c\_2Enum\_2E0) \wedge \\
(V1m = c\_2Enum\_2E0))) \wedge (((ap\ c\_2Ereal\_2Ereal\_of\_num\ V0n) = \\
(ap\ c\_2Erealax\_2Ereal\_neg\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ V1m))) \Leftrightarrow \\
((V0n = c\_2Enum\_2E0) \wedge (V1m = c\_2Enum\_2E0))) \wedge (((ap\ c\_2Erealax\_2Ereal\_neg \\
(ap\ c\_2Ereal\_2Ereal\_of\_num\ V0n)) = (ap\ c\_2Erealax\_2Ereal\_neg \\
(ap\ c\_2Ereal\_2Ereal\_of\_num\ V1m))) \Leftrightarrow (V0n = V1m)))))) \\
& \hspace{15em} (45)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0m \in ty\_2Einteger\_2Eint. ((ap\ c\_2Eintreal\_2Ereal\_of\_int \\
& \quad (ap\ c\_2Einteger\_2Eint\_neg\ V0m)) = (ap\ c\_2Erealax\_2Ereal\_neg \\
& \quad \quad (ap\ c\_2Eintreal\_2Ereal\_of\_int\ V0m))))
\end{aligned}$$