

thm_2Eintreal_2Ereal_of_int_num
(TMKVWvUf6CJzsn7m9L7k9hqo6uax1hYxZhx)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{3}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)})ty_2Einteger_2Eint) \tag{4}$$

Definition 9 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E40 (ty_2Einteger_2Eint_neg) a))$. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum) (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)) \quad (5)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)}) (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)})} \quad (7)$$

Definition 10 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)$.

Definition 11 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint_neg T1)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Eenum_2Eenum} \quad (8)$$

Definition 12 We define $c_2Einteger_2Eenum$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap (c_2Emin_2E40 ty_2Eenum) i)$.

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)}) (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)) \quad (9)$$

Definition 13 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint_lt T1 T2)$.

Definition 14 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21) t1 t2)))$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap c_2Ebool_2E21 t1 t2))))$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (10)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Eenum_2Eenum} \quad (11)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) ty_2Erealax_2Ereal) \quad (13)$$

Definition 16 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E.40 (t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (16)$$

Definition 17 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 18 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (18)$$

Definition 19 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 20 We define $c_2Eintreal_2Ereal_of_int$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap (ap (ap (c_2Ebo$

Definition 21 We define $c_2Ebool_2E.7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E.3D_3D_3E V0t) c_2Ebool_2E$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega)^{ty_2Enum_2Enum} \quad (19)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega)^{\omega} \quad (20)$$

Definition 22 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 23 We define $c_2Eprim_rec_2E.3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg (p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27))))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & ((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\ & V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
V0n)) (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V0n) V1m))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
(ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_neg \\
(ap c_2Einteger_2Eint_of_num V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V1m) V0n))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
(ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_of_num \\
V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0)))) \wedge ((p \\
(ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
V0n)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
V1m)))) \Leftrightarrow False))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. ((\exists V1n \in ty_2Enum_2Enum. \\
((V0p = (ap c_2Einteger_2Eint_of_num V1n)) \wedge (\neg(V1n = c_2Enum_2E0)))) \vee \\
((\exists V2n \in ty_2Enum_2Enum. ((V0p = (ap c_2Einteger_2Eint_neg \\
(ap c_2Einteger_2Eint_of_num V2n)) \wedge (\neg(V2n = c_2Enum_2E0)))) \vee \\
(V0p = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))))))
\end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Einteger_2Eint (ap c_2Einteger_2Eint_of_num \\
V0n)) = V0n)) \tag{34}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
(((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\
V1n)) \Leftrightarrow (V0m = V1n))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. (\forall V3y \in \\
ty_2Einteger_2Eint. (((ap c_2Einteger_2Eint_neg V2x) = (ap c_2Einteger_2Eint_neg \\
V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty_2Enum_2Enum. (\forall V5m \in \\
ty_2Enum_2Enum. (((ap c_2Einteger_2Eint_of_num V4n) = (ap \\
c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V5m))) \Leftrightarrow \\
((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap c_2Einteger_2Eint_neg \\
(ap c_2Einteger_2Eint_of_num V4n)) = (ap c_2Einteger_2Eint_of_num \\
V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0)))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty_2Einteger_2Eint. ((ap\ c_2Eintreal_2Ereal_of_int \\
& V0i) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal)\ (ap\ (ap \\
& c_2Einteger_2Eint_lt\ V0i)\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)))) \\
& (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ (ap \\
& c_2Einteger_2ENum\ (ap\ c_2Einteger_2Eint_neg\ V0i))))))\ (ap\ c_2Ereal_2Ereal_of_num \\
& (ap\ c_2Einteger_2ENum\ V0i))))
\end{aligned} \tag{36}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ c_2Enum_2E0)))) \tag{37}$$

Assume the following.

$$\begin{aligned}
& ((ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (((ap\ c_2Ereal_2Ereal_of_num\ V0n) = (ap\ c_2Ereal_2Ereal_of_num \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num \\
& V0n)) = (ap\ c_2Ereal_2Ereal_of_num\ V1m)) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge \\
& (V1m = c_2Enum_2E0))) \wedge (((ap\ c_2Ereal_2Ereal_of_num\ V0n) = \\
& (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V1m))) \Leftrightarrow \\
& ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap\ c_2Erealax_2Ereal_neg \\
& (ap\ c_2Ereal_2Ereal_of_num\ V0n)) = (ap\ c_2Erealax_2Ereal_neg \\
& (ap\ c_2Ereal_2Ereal_of_num\ V1m))) \Leftrightarrow (V0n = V1m))))))
\end{aligned} \tag{39}$$

Theorem 1

$$(\forall V0n \in ty_2Enum_2Enum. ((ap\ c_2Eintreal_2Ereal_of_int\ (ap\ c_2Einteger_2Eint_of_num\ V0n)) = (ap\ c_2Ereal_2Ereal_of_num\ V0n)))$$