

thm\_2Eintto\_2Eneg\_\_BIT2\_\_lt\_\_thm  
(TMJiF8dQgGRJbhWMUhfFPxjMuGoauPSCCWpL)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda P \in 2^A.(ap P (ap (c\_2Emin\_2E\_40 P)))$

**Definition 12** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (5)$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty ty\_2Einteger\_2Eint \quad (6)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \quad (7)$$

**Definition 13** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40 (ty\_2Einteger\_2Eint\_REP\_CLASS a)))$

Let  $c\_2Einteger\_2Eint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_neg \in ((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{ty\_2Einteger\_2Eint}) \quad (8)$$

Let  $c\_2Einteger\_2Eint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)}) \quad (9)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})} \quad (10)$$

**Definition 14** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum).$

**Definition 15** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap c\_2Einteger\_2Eint\_neg T1)$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint)^{ty\_2Enum\_2Enum} \quad (11)$$

Let  $c\_2Einteger\_2Eint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)}) \quad (12)$$

**Definition 16** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint.$

Let  $ty\_2EternaryComparisons\_2Eordering : \iota$  be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \quad (13)$$

Let  $c\_2EternaryComparisons\_2EGREATER : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \quad (14)$$

Let  $c\_2EternaryComparisons\_2ELESS : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \quad (15)$$

**Definition 17** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

Let  $c\_2EternaryComparisons\_2EEQUAL : \iota$  be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \quad (16)$$

**Definition 18** We define  $c\_2Etoto\_2ETO\_of\_LinearOrder$  to be  $\lambda A\_27a : \iota. \lambda V0r \in ((2^{A\_27a})^{A\_27a}). \lambda V1x \in$

**Definition 19** We define  $c\_2Eintto\_2EintOrd$  to be  $(ap\ (c\_2Etoto\_2ETO\_of\_LinearOrder\ ty\_2Einteger\_2Ei$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (17)$$

**Definition 20** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 21** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \\
& V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ V1Q) \ V3x\_27) \\
& V5y\_27))))))))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in \\
& A\_27a.((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap \\
& (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \ V2t1) \ V3t2) = V3t2)))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num \\
V0n)) (ap c\_2Einteger\_2Eint\_of\_num V1m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num V0n)) (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num V1m)))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V1m) V0n))) \wedge (((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num V0n)) (ap c\_2Einteger\_2Eint\_of\_num \\
V1m))) \Leftrightarrow ((\neg(V0n = c\_2Enum\_2E0)) \vee (\neg(V1m = c\_2Enum\_2E0)))) \wedge ((p \\
& (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num \\
V0n)) (ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num \\
V1m)))) \Leftrightarrow False))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Einteger\_2Eint\_of\_num V0m) = (ap c\_2Einteger\_2Eint\_of\_num \\
V1n)) \Leftrightarrow (V0m = V1n))) \wedge ((\forall V2x \in ty\_2Einteger\_2Eint. (\forall V3y \in \\
& ty\_2Einteger\_2Eint. (((ap c\_2Einteger\_2Eint\_neg V2x) = (ap c\_2Einteger\_2Eint\_neg \\
V3y)) \Leftrightarrow (V2x = V3y)))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. (\forall V5m \in \\
& ty\_2Enum\_2Enum. (((ap c\_2Einteger\_2Eint\_of\_num V4n) = (ap \\
c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num V5m))) \Leftrightarrow \\
& ((V4n = c\_2Enum\_2E0) \wedge (V5m = c\_2Enum\_2E0))) \wedge (((ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num V4n)) = (ap c\_2Einteger\_2Eint\_of\_num \\
V5m)) \Leftrightarrow ((V4n = c\_2Enum\_2E0) \wedge (V5m = c\_2Enum\_2E0))))))
\end{aligned} \tag{31}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg((ap c\_2Earithmetic\_2EBIT2 V0n) = c\_2Enum\_2E0))) \tag{32}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Eintto\_2EintOrd (ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2EBIT2 V0m)))) (ap c\_2Einteger\_2Eint\_of\_num \\
V1n)) = c\_2EternaryComparisons\_2ELESS)))
\end{aligned}$$