

thm_2Eintto_2Eneg_ZERO_eq_ZERO_thm
 (TMM-
 rLeD8t3B3D9skDw9wTCmJLyMZFgh54x3)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p \ Q)$ of type ι .

Definition 5 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2. V0x))\ (\lambda V1x \in 2. V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1t \in 2. V1t))\ (\lambda V2t \in 2. V2t)))$

Definition 7 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2. V0t))$.

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ (c_2Emin_2E_3D_3D_3E\ V0t))\ c_2Ebool_2EF))$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V2t \in 2. V2t)))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V2t \in 2. V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\lambda x.x \in A \wedge P(x)) \text{ else } \perp$

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Einteger_2Eint \quad (7)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (8)$$

Definition 15 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)))$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (9)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (10)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (11)$$

Definition 16 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum).$

Definition 17 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (13)$$

Definition 18 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger_2Eint. \dots$

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (14)$$

Let $c_2EternaryComparisons_2EGREATER : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EGREATER \in ty_2EternaryComparisons_2Eordering \quad (15)$$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \quad (16)$$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (\dots)))$

Let $c_2EternaryComparisons_2EEQUAL : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EEQUAL \in ty_2EternaryComparisons_2Eordering \quad (17)$$

Definition 20 We define $c_2Etoto_2ETO_of_LinearOrder$ to be $\lambda A_27a : \iota. \lambda V0r \in ((2^{A_27a})^{A_27a}). \lambda V1x \in$

Definition 21 We define $c_2Eintto_2EintOrd$ to be $(ap\ (c_2Etoto_2ETO_of_LinearOrder\ ty_2Einteger_2Eint))$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \quad (22) \\ ((\neg False) \Leftrightarrow True)))$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \quad (23) \\ True))$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \quad (24) \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \quad (25) \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0t1 \in A_27a.(\forall V1t2 \in \quad (27) \\ & A_27a.((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a.(\forall V3t2 \in A_27a.((ap \\ & (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& V0n)) (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_of_num \\
& V1m)))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0))) \wedge ((p \\
& (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& V0n)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& V1m)))) \Leftrightarrow False)))))) \\
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\
& V1n)) \Leftrightarrow (V0m = V1n))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. (\forall V3y \in \\
& ty_2Einteger_2Eint. (((ap c_2Einteger_2Eint_neg V2x) = (ap c_2Einteger_2Eint_neg \\
& V3y)) \Leftrightarrow (V2x = V3y))) \wedge (\forall V4n \in ty_2Enum_2Enum. (\forall V5m \in \\
& ty_2Enum_2Enum. (((ap c_2Einteger_2Eint_of_num V4n) = (ap \\
& c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V5m)) \Leftrightarrow \\
& ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V4n)) = (ap c_2Einteger_2Eint_of_num \\
& V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))) \\
\end{aligned} \tag{29}$$

Theorem 1

$$\begin{aligned}
& ((ap (ap c_2Eintto_2EintOrd (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& c_2Earithmetic_2EZERO))) (ap c_2Einteger_2Eint_of_num c_2Earithmetic_2EZERO)) = \\
& c_2EternaryComparisons_2EQUAL)
\end{aligned}$$