

thm_2Eintto_2Eneg_lt_BIT2_thm (TM- NTtVaGoHiNVFWriVYjyS477XFbcYhpZvd)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 10 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A) \text{ of type } \iota \Rightarrow \iota$.

Definition 11 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40$

Definition 12 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}$

Let $\text{ty_2Epair_2Eprod} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty_2Epair_2Eprod } A0 \ A1) \quad (5)$$

Let $\text{ty_2Einteger_2Eint} : \iota$ be given. Assume the following.

$$\text{nonempty ty_2Einteger_2Eint} \quad (6)$$

Let $\text{c_2Einteger_2Eint_REP_CLASS} : \iota$ be given. Assume the following.

$$\text{c_2Einteger_2Eint_REP_CLASS} \in ((2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \text{ty_2Einteger_2Eint}})) \quad (7)$$

Definition 13 We define `c_2Einteger_2Eint_REP` to be $\lambda V0a \in \text{ty_2Einteger_2Eint}. (\text{ap (c_2Emin_2E_40 (t$

Let $\text{c_2Einteger_2Etint_neg} : \iota$ be given. Assume the following.

$$\text{c_2Einteger_2Etint_neg} \in ((\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum})^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum})}) \quad (8)$$

Let $\text{c_2Einteger_2Etint_eq} : \iota$ be given. Assume the following.

$$\text{c_2Einteger_2Etint_eq} \in ((2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \text{ty_2Epair_2Eprod ty_2Enum_2Enum}})) \quad (9)$$

Let $\text{c_2Einteger_2Eint_ABS_CLASS} : \iota$ be given. Assume the following.

$$\text{c_2Einteger_2Eint_ABS_CLASS} \in (\text{ty_2Einteger_2Eint}^{(2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \text{ty_2Enum_2Enum}})}) \quad (10)$$

Definition 14 We define `c_2Einteger_2Eint_ABS` to be $\lambda V0r \in (\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum})$

Definition 15 We define `c_2Einteger_2Eint_neg` to be $\lambda V0T1 \in \text{ty_2Einteger_2Eint}. (\text{ap c_2Einteger_2Eint_neg (c_2Einteger_2Eint_neg$

Let $\text{c_2Einteger_2Eint_of_num} : \iota$ be given. Assume the following.

$$\text{c_2Einteger_2Eint_of_num} \in (\text{ty_2Einteger_2Eint}^{\text{ty_2Enum_2Enum}}) \quad (11)$$

Let $\text{c_2Einteger_2Etint_lt} : \iota$ be given. Assume the following.

$$\text{c_2Einteger_2Etint_lt} \in ((2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \text{ty_2Epair_2Eprod ty_2Enum_2Enum}})) \quad (12)$$

Definition 16 We define `c_2Einteger_2Eint_lt` to be $\lambda V0T1 \in \text{ty_2Einteger_2Eint}. \lambda V1T2 \in \text{ty_2Einteger_2Eint}$

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (13)$$

Let $c_2EternaryComparisons_2EGREATER : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EGREATER \in ty_2EternaryComparisons_2Eordering \quad (14)$$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \quad (15)$$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_2EternaryComparisons_2EEQUAL : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EEQUAL \in ty_2EternaryComparisons_2Eordering \quad (16)$$

Definition 18 We define $c_2Etoto_2ETO_of_LinearOrder$ to be $\lambda A_27a : \iota. \lambda V0r \in ((2^{A_27a})^{A_27a}). \lambda V1x \in$

Definition 19 We define $c_2Eintto_2EintOrd$ to be $(ap\ (c_2Etoto_2ETO_of_LinearOrder\ ty_2Einteger_2Ei$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Definition 20 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 21 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\
& V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\
& V5y_27))))))))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a.(\forall V3t2 \in A_27a.((ap \\
& (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \ V2t1) \ V3t2) = V3t2)))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
V0n)) (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V1m) V0n))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_of_num \\
V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0)))) \wedge ((p \\
& (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
V0n)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
V1m)))) \Leftrightarrow False))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\
V1n)) \Leftrightarrow (V0m = V1n))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. (\forall V3y \in \\
& ty_2Einteger_2Eint. (((ap c_2Einteger_2Eint_neg V2x) = (ap c_2Einteger_2Eint_neg \\
V3y)) \Leftrightarrow (V2x = V3y)))) \wedge ((\forall V4n \in ty_2Enum_2Enum. (\forall V5m \in \\
& ty_2Enum_2Enum. (((ap c_2Einteger_2Eint_of_num V4n) = (ap \\
c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V5m))) \Leftrightarrow \\
& ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V4n)) = (ap c_2Einteger_2Eint_of_num \\
V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))
\end{aligned} \tag{31}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap c_2Earithmetic_2EBIT2 V0n) = c_2Enum_2E0))) \tag{32}$$

Theorem 1

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Eintto_2EintOrd (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
V0m))) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2EBIT2 \\
V1n))) = c_2EternaryComparisons_2ELESS)))
\end{aligned}$$