

thm_2Eintto_2Epos__pos__thm
(TMQYS2DCW3qojwogRNXYZNzSSkcb6A5o4u)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 6 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_EF))$

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{4}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (5)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (6)$$

Definition 9 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E40\ (t$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (7)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (8)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (9)$$

Definition 11 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 12 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (10)$$

Definition 13 We define c_2Ebool_2E2F5C to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (11)$$

Definition 14 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $ty_2EternaryComparisons_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty_2EternaryComparisons_2Eordering \quad (12)$$

Let $c_2EternaryComparisons_2EGREATER : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2EGREATER \in ty_2EternaryComparisons_2Eordering \quad (13)$$

Let $c_2EternaryComparisons_2ELESS : \iota$ be given. Assume the following.

$$c_2EternaryComparisons_2ELESS \in ty_2EternaryComparisons_2Eordering \quad (14)$$

Definition 15 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_EternaryComparisons_EEQUAL : \iota$ be given. Assume the following.

$$c_EternaryComparisons_EEQUAL \in ty_EternaryComparisons_Eordering \quad (15)$$

Definition 16 We define $c_Etoto_ETO_of_LinearOrder$ to be $\lambda A_27a : \iota. \lambda V0r \in ((2^{A_27a})^{A_27a}). \lambda V1x \in$

Definition 17 We define $c_Eintto_EintOrd$ to be $(ap (c_Etoto_ETO_of_LinearOrder ty_Einteger_EintOrd$

Let $c_Enum_EREP_num : \iota$ be given. Assume the following.

$$c_Enum_EREP_num \in (omega^{ty_Enum_Enum}) \quad (16)$$

Let $c_Enum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Enum_ESUC_REP \in (omega^{omega}) \quad (17)$$

Definition 18 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum. (ap c_Enum_EABS_num$

Definition 19 We define $c_Ebool_E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_E_40$

Definition 20 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_Enum_Enum. \lambda V1n \in ty_Enum_Enum$

Definition 21 We define $c_Etoto_EnumOrd$ to be $(ap (c_Etoto_ETO_of_LinearOrder ty_Enum_Enum$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg (p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_Ebool_ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_Ebool_ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad V1n)) \Leftrightarrow (V0m = V1n)))) \\
& \tag{23}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\
& V0n))\ (ap\ c_2Einteger_2Eint_of_num\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_neg \\
& (ap\ c_2Einteger_2Eint_of_num\ V0n))\ (ap\ c_2Einteger_2Eint_neg \\
& (ap\ c_2Einteger_2Eint_of_num\ V1m)))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V1m)\ V0n))) \wedge (((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_neg \\
& (ap\ c_2Einteger_2Eint_of_num\ V0n))\ (ap\ c_2Einteger_2Eint_of_num \\
& V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0)))) \wedge ((p \\
& (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\
& V0n))\ (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\
& V1m)))) \Leftrightarrow False)))))) \\
& \tag{24}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap\ (ap\ c_2Eintto_2EintOrd\ (ap\ c_2Einteger_2Eint_of_num\ V0m)) \\
& (ap\ c_2Einteger_2Eint_of_num\ V1n)) = (ap\ (ap\ c_2Etoto_2EnumOrd \\
& \quad V0m)\ V1n))))
\end{aligned}$$