thm_2Eiterate_2EFINITE__RECURSION (TMR-phmaYAWzM4T23dr5hoGBfMhMvuya4RXE)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 6 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 9 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 11 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 13 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b\in((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})\tag{3}$$

Definition 14 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a:\iota.\lambda V0x\in A\_27a.\lambda V1s\in(2^{A\_27a}).(ap\ (c\_$

Definition 15 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a:\iota.(\lambda V0x\in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 16 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (2$

Definition 17 *We define* c_2Eiterate_2EITSET *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

Assume the following.

$$True\tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t\in 2.((\forall V1x\in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t)))\tag{5}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))\tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f\in((A\_27b^{A\_27b})^{A\_27a}).(\forall V1b\in A\_27b.((\forall V2x\in A\_27a.(\forall V3y\in A\_27a.(\forall V4s\in A\_27b.((\neg(V2x=V3y))\Rightarrow((ap\ (ap\ V0f\ V2x)\ (ap\ (ap\ V0f\ V3y)\ V4s))=(ap\ (ap\ V0f\ V3y)\ (ap\ (ap\ V0f\ V2x)\ V4s)))))))\Rightarrow(\exists V5g\in(A\_27b^{(2^{A\_27a})}).(((ap\ V5g\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))=V1b)\wedge(\forall V6x\in A\_27a.(\forall V7s\in(2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V7s))\Rightarrow((ap\ V5g\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V6x)\ V7s))=(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6x)\ V7s))\ (ap\ V5g\ V7s))\ (ap\ (ap\ V0f\ V6x)\ (ap\ V5g\ V7s)))))))))))))\tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V1b \in A\_27b.((\forall V2x \in A\_27a.(\forall V3y \in A\_27a.(\forall V4s \in A\_27b.((\neg(V2x = V3y)) \Rightarrow ((ap\ (ap\ V0f\ V2x)\ (ap\ (ap\ V0f\ V3y)\ V4s)) = (ap\ (ap\ V0f\ V3y)\ (ap\ (ap\ V0f\ V2x)\ V4s))))))) \Rightarrow (((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2EITSET}\ A\_27b\ A\_27a)\ V0f)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ V1b) = V1b) \land (\forall V5x \in A\_27a.(\forall V6s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V6s)) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2EITSET}\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V5x)\ V6s))\ V1b) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5x)\ V6s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2EITSET}\ A\_27b\ A\_27a)\ V0f)\ V6s)\ V1b))\ (ap\ (ap\ V0f\ V5x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2EITSET}\ A\_27b\ A\_27a)\ V0f)\ V6s)\ V1b)))))))))))$$