

thm_2Eiterate_2EFINREC_def_compute (TM-SkiR5tmCL7NLNxZ9wW9V9mu2FBW3E33JR)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda P \in (2^{A-27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V0P \in 2.V0P)))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 6 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2 n) V0)$

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let c_2 be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum^{ty_2Enum_2Enum}})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 8 We define `c_2Earthmetic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earthmetic_2EBIT1\ n)\ V)$

Definition 9 We define `c_2Earthmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 10 We define $c_{\text{2Emin_2E_3D_3D_3E}}$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o} (p \Rightarrow p \ Q)$ of type ι .

Definition 11 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2E_{pred_set_2EEMPTY}$ to be $\lambda A._27a : \iota.(\lambda V0x \in A._27a.c_2Ebool_2EF)$.

Definition 13 We define $c_{\text{Ebool_2EIN}}$ to be $\lambda A.\lambda 27a:\iota.(\lambda V0x \in A.27a).(\lambda V1f \in (2^A.27a)).(ap\;V1f\;V0x)$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in$

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty}(\text{ty_2Epair_2Eprod } A0\ A1) \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{\text{27}}.a.\text{nonempty } A_{\text{27}}a \Rightarrow \forall A_{\text{27}}b.\text{nonempty } A_{\text{27}}b \Rightarrow c_{\text{2Epair_2EABS_prod}}(A_{\text{27}}a, A_{\text{27}}b) \in ((ty_{\text{2Epair_2Eprod}}(A_{\text{27}}a, A_{\text{27}}b))^{\langle (2^{A_{\text{27}}b})^{A_{\text{27}}a} \rangle}) \quad (9)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2\text{Epred_set_2EGSPEC } A_27a \ A_27b \in ((2^{A_27a})^{((ty_2\text{Epair_2Eprod } A_27a \ 2)^{A_27b})}) \quad (10)$$

Definition 17 We define $c_2Epred_set_2EINSERT$ to be $\lambda A.\lambda a : \iota. \lambda V0x \in A. \lambda 27a. \lambda V1s \in (2^{A \rightarrow 27a}). (ap (c_2Epred_set) (cons a s))$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Definition 19 We define $c_2\text{EPred_set_2EDIFF}$ to be $\lambda A.\lambda 27a:\iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap(c_2\text{EPred_set_2EDIFF})(V0,V1)) = t$

Definition 20 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap (ap$

Definition 21 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge P(x)) \text{ else } \iota$

Definition 22 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Let $c_2Eiterate_2EFINREC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eiterate_2EFINREC \\ & A_27a A_27b \in (((((2^{ty_2Enum_2Enum})^{A_27b})^{(2^{A_27a})}))^{A_27b})^{((A_27b)^{A_27b})^{A_27a}}) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0f \in ((A_27a)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\ & (\forall V1g \in (A_27a)^{ty_2Enum_2Enum}). ((\forall V2n \in ty_2Enum_2Enum. \\ & ((ap V1g (ap c_2Enum_2ESUC V2n)) = (ap (ap V0f V2n) (ap c_2Enum_2ESUC \\ & V2n))) \Leftrightarrow ((\forall V3n \in ty_2Enum_2Enum. ((ap V1g (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 V3n)) = (ap (ap V0f (ap (ap c_2Earithmetic_2E_2D \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n)))) \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 V3n)))) \wedge \\ & (\forall V4n \in ty_2Enum_2Enum. ((ap V1g (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 V4n)) = (ap (ap V0f (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 V4n))) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 V4n)))))))))) \end{aligned} \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow \\ & (\forall V0f \in ((A_27b)^{A_27b})^{A_27a}). (\forall V1b \in A_27b. (\forall V2s \in \\ & (2^{A_27a}). (\forall V3a \in A_27b. ((p (ap (ap (ap (ap (c_2Eiterate_2EFINREC \\ & A_27a A_27b) V0f) V1b) V2s) V3a) c_2Enum_2E0)) \Leftrightarrow (V2s = (c_2Epred_set_2EEMPTY \\ & A_27a)) \wedge (V3a = V1b))))))) \wedge (\forall V4f \in ((A_27b)^{A_27b})^{A_27a}). \\ & (\forall V5b \in A_27b. (\forall V6s \in (2^{A_27a}). (\forall V7a \in A_27b. \\ & (\forall V8n \in ty_2Enum_2Enum. ((p (ap (ap (ap (ap (c_2Eiterate_2EFINREC \\ & A_27a A_27b) V4f) V5b) V6s) V7a) (ap c_2Enum_2ESUC V8n)) \Leftrightarrow (\exists V9x \in \\ & A_27a. (\exists V10c \in A_27b. ((p (ap (ap (c_2Ebool_2EIN A_27a) V9x) \\ & V6s)) \wedge ((p (ap (ap (ap (ap (c_2Eiterate_2EFINREC A_27a A_27b) \\ & V4f) V5b) (ap (ap (c_2Epred_set_2EDELETE A_27a) V6s) V9x)) V10c) \\ & V8n)) \wedge (V7a = (ap (ap V4f V9x) V10c))))))))))))))) \end{aligned} \quad (14)$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \\
& (\forall V0f \in ((A_{.27b}^{A_{.27b}})^{A_{.27a}}).(\forall V1b \in A_{.27b}.(\forall V2s \in \\
& (2^{A_{.27a}}).(\forall V3a \in A_{.27b}.((p\ (ap\ (ap\ (ap\ (ap\ (c_{.2Eiterate_2EFINREC} \\
& A_{.27a}\ A_{.27b})\ V0f)\ V1b)\ V2s)\ V3a)\ c_{.2Enum_2E0})) \Leftrightarrow ((V2s = (c_{.2Epred_set_2EEMPTY} \\
& A_{.27a})) \wedge (V3a = V1b))))))) \wedge ((\forall V4f \in ((A_{.27b}^{A_{.27b}})^{A_{.27a}}). \\
& (\forall V5b \in A_{.27b}.(\forall V6s \in (2^{A_{.27a}}).(\forall V7a \in A_{.27b}. \\
& (\forall V8n \in ty_{.2Enum_2Enum}.((p\ (ap\ (ap\ (ap\ (ap\ (c_{.2Eiterate_2EFINREC} \\
& A_{.27a}\ A_{.27b})\ V4f)\ V5b)\ V6s)\ V7a)\ (ap\ c_{.2Earithmetic_2ENUMERAL} (\\
& ap\ c_{.2Earithmetic_2EBIT1}\ V8n)))))) \Leftrightarrow (\exists V9x \in A_{.27a}.(\exists V10c \in \\
& A_{.27b}.((p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V9x)\ V6s)) \wedge ((p\ (ap\ (ap\ (\\
& ap\ (ap\ (c_{.2Eiterate_2EFINREC}\ A_{.27a}\ A_{.27b})\ V4f)\ V5b)\ (ap\ (ap\ (\\
& c_{.2Epred_set_2EDELETE}\ A_{.27a})\ V6s)\ V9x))\ V10c)\ (ap\ (ap\ c_{.2Earithmetic_2E_2D} \\
& (ap\ c_{.2Earithmetic_2ENUMERAL}\ (ap\ c_{.2Earithmetic_2EBIT1}\ V8n)))) \\
& (ap\ c_{.2Earithmetic_2ENUMERAL}\ (ap\ c_{.2Earithmetic_2EBIT1}\ c_{.2Earithmetic_2EZERO})))))) \wedge \\
& (V7a = (ap\ (ap\ V4f\ V9x)\ V10c)))))))))) \wedge ((\forall V11f \in ((A_{.27b}^{A_{.27b}})^{A_{.27a}}). \\
& (\forall V12b \in A_{.27b}.(\forall V13s \in (2^{A_{.27a}}).(\forall V14a \in \\
& A_{.27b}.(\forall V15n \in ty_{.2Enum_2Enum}.((p\ (ap\ (ap\ (ap\ (ap\ (c_{.2Eiterate_2EFINREC} \\
& A_{.27a}\ A_{.27b})\ V11f)\ V12b)\ V13s)\ V14a)\ (ap\ c_{.2Earithmetic_2ENUMERAL} \\
& (ap\ c_{.2Earithmetic_2EBIT2}\ V15n)))))) \Leftrightarrow (\exists V16x \in A_{.27a}.(\exists V17c \in \\
& A_{.27b}.((p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V16x)\ V13s)) \wedge ((p\ (ap\ (ap\ (\\
& ap\ (ap\ (c_{.2Eiterate_2EFINREC}\ A_{.27a}\ A_{.27b})\ V11f)\ V12b)\ (ap\ (\\
& ap\ (c_{.2Epred_set_2EDELETE}\ A_{.27a})\ V13s)\ V16x))\ V17c)\ (ap\ c_{.2Earithmetic_2ENUMERAL} \\
& (ap\ c_{.2Earithmetic_2EBIT1}\ V15n)))))) \wedge (V14a = (ap\ (ap\ V11f\ V16x)\ V17c)))))))))))))))
\end{aligned}$$