thm_2Eiterate_2EITERATE__CLAUSES__GEN (TMbEqPXPx31tCsKYk1UnjK24hzgHboVivbR)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}\ A$

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 6 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 7 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\text{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 11 *We define* c_2Eiterate_2Eneutral *to be* $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (\text{c\_2Emin}$

Definition 12 *We define* c_2Eiterate_2Emonoidal *to be* $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (ap\ \text{c\_2}$

Definition 13 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 17 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 18 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 19 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 20 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 21 *We define* c_2Eiterate_2EITSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

Definition 22 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 23 *We define* c_2Eiterate_2Esupport *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 24 *We define* c_2Eiterate_2Eiterate *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 25 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 26 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 27 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 28 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 29 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 30 *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{13}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in 2.(\forall V1t \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0b)\ V1t)\ V1t) = V1t))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \vee (p\ V1Q)) \Leftrightarrow (\exists V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \vee (\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \forall A\_27b.\mathit{nonempty}\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. (\forall V3y \in A\_27a.((ap\ V0f\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ V1b)\ (ap\ V0f\ V2x))\ (ap\ V0f\ V3y)))))))\tag{33}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27))))))))\tag{34}$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. (\forall V5y\_27 \in A\_27a.((((p\ V0P)\Leftrightarrow(p\ V1Q))\wedge(((p\ V1Q)\Rightarrow(V2x = V3x\_27))\wedge ((\neg(p\ V1Q))\Rightarrow(V4y = V5y\_27))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27)))))))))\tag{35}$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \forall A\_27b.\mathit{nonempty}\ A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y))))\Leftrightarrow(\exists V3f \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))))\tag{36}$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x))\tag{37}$$

Assume the following.

$$\forall A\_27a.\mathit{nonempty}\ A\_27a \Rightarrow \forall A\_27b.\mathit{nonempty}\ A\_27b \Rightarrow (\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V1b \in A\_27b.((\forall V2x \in A\_27a.(\forall V3y \in A\_27a.(\forall V4s \in A\_27b.((\neg(V2x = V3y))\Rightarrow ((ap\ (ap\ V0f\ V2x)\ (ap\ (ap\ V0f\ V3y)\ V4s)) = (ap\ (ap\ V0f\ V3y)\ (ap\ (ap\ V0f\ V2x)\ V4s)))))))\Rightarrow(((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2EITSET}\ A\_27b\ A\_27a)\ V0f)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ V1b) = V1b)\wedge(\forall V5x \in A\_27a.(\forall V6s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V6s))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2EITSET}\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V5x)\ V6s))\ V1b) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5x)\ V6s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2EITSET}\ A\_27b\ A\_27a)\ V0f)\ V6s)\ V1b))\ (ap\ (ap\ V0f\ V5x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2EITSET}\ A\_27b\ A\_27a)\ V0f)\ V6s)\ V1b)))))))))))\tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V1f \in (A\_27a^{A\_27b}). (\forall V2x \in A\_27b.(\forall V3s \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V2x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27b\ A\_27a)\ V0op)\ V1f)\ V3s))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V2x)\ V3s)) \wedge (\neg((ap\ V1f\ V2x) = (ap\ (\mathsf{c\_2Eiterate\_2Eneutral}\ A\_27a)\ V0op)))))))))) \quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow \forall A\_27g.nonempty\ A\_27g \Rightarrow \forall A\_27h.nonempty\ A\_27h \Rightarrow \forall A\_27i.nonempty\ A\_27i \Rightarrow (\forall V0op \in ((A\_27b^{A\_27b})^{A\_27b}).((\forall V1f \in (A\_27b^{A\_27a}). ((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27a\ A\_27b)\ V0op)\ V1f)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \wedge ((\forall V2f \in (A\_27b^{A\_27c}). (\forall V3x \in A\_27c.(\forall V4s \in (2^{A\_27c}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27c\ A\_27b)\ V0op)\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27c)\ V3x)\ V4s)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (2^{A\_27c}))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27b)\ (ap\ V2f\ V3x))\ (ap\ (\mathsf{c\_2Eiterate\_2Eneutral}\ A\_27b)\ V0op)))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27c\ A\_27b)\ V0op)\ V2f)\ V4s))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27c)\ V3x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27c\ A\_27b)\ V0op)\ V2f)\ V4s))))))) \wedge ((\forall V5f \in (A\_27b^{A\_27d}). (\forall V6x \in A\_27d.(\forall V7s \in (2^{A\_27d}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27d\ A\_27b)\ V0op)\ V5f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27d)\ V7s)\ V6x)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27d)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27d\ A\_27b)\ V0op)\ V5f)\ V7s))\ V6x))))) \wedge ((\forall V8f \in (A\_27b^{A\_27e}).(\forall V9s \in (2^{A\_27e}).(\forall V10t \in (2^{A\_27e}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27e\ A\_27b)\ V0op)\ V8f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27e)\ V9s)\ V10t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27e)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27e\ A\_27b)\ V0op)\ V8f)\ V9s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27e\ A\_27b)\ V0op)\ V8f)\ V10t)))))) \wedge ((\forall V11f \in (A\_27b^{A\_27f}). (\forall V12s \in (2^{A\_27f}).(\forall V13t \in (2^{A\_27f}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27f\ A\_27b)\ V0op)\ V11f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27f)\ V12s)\ V13t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27f)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27f\ A\_27b)\ V0op)\ V11f)\ V12s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27f\ A\_27b)\ V0op)\ V11f)\ V13t)))))) \wedge ((\forall V14f \in (A\_27b^{A\_27g}).(\forall V15s \in (2^{A\_27g}).(\forall V16t \in (2^{A\_27g}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27g\ A\_27b)\ V0op)\ V14f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27g)\ V15s)\ V16t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27g)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27g\ A\_27b)\ V0op)\ V14f)\ V15s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27g\ A\_27b)\ V0op)\ V14f)\ V16t)))))) \wedge (\forall V17f \in (A\_27i^{A\_27h}). (\forall V18g \in (A\_27b^{A\_27i}).(\forall V19s \in (2^{A\_27h}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27i\ A\_27b)\ V0op)\ V18g)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27h\ A\_27i)\ V17f)\ V19s)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27h\ A\_27i)\ V17f)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27h\ A\_27b)\ V0op)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27h\ A\_27b\ A\_27i)\ V18g)\ V17f))\ V19s)))))))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ V1s))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s))))) \tag{42}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{43}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{45}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{52}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0op \in ((A\_27b^{A\_27b})^{A\_27b}).((p\ (ap\ (\mathsf{c\_2Eiterate\_2Emonoidal}\ A\_27b)\ V0op))\Rightarrow((\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27b)\ V0op)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ V1f) = (ap\ (\mathsf{c\_2Eiterate\_2Eneutral}\ A\_27b)\ V0op)))\wedge(\forall V2f \in (A\_27b^{A\_27a}).(\forall V3x \in A\_27a. (\forall V4s \in (2^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Eiterate\_2Emonoidal}\ A\_27b)\ V0op))\wedge(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27a\ A\_27b)\ V0op)\ V2f)\ V4s))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27b)\ V0op)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V3x)\ V4s))\ V2f) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V4s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27b)\ V0op)\ V4s)\ V2f))\ (ap\ (ap\ V0op\ (ap\ V2f\ V3x))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27b)\ V0op)\ V4s)\ V2f)))))))))))$$