

thm_2Eiterate_2EITERATE__DELTA (TMd- WCPZKEcsQcyAZsirU7CHuumVDCoopYVj)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 8 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}).$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod V0x V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (3)$$

Definition 12 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 13 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 14 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 15 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 17 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2E$

Definition 18 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap\ (ap$

Definition 19 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge P\ x)$
of type $\iota \Rightarrow \iota$.

Definition 20 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ (c_2Emin$

Definition 21 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 22 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 23 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2$

Definition 24 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$

Definition 25 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 26 We define $c_2Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ (ap\ c_2E$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap (ap (ap (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a.(\forall V5y_27 \in A_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap (ap (ap (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow (\\
& \forall V0op \in ((A.27b^{A.27b})^{A.27b}).((\forall V1f \in (A.27b^{A.27a}). \\
((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27a\ A.27b)\ V0op)\ V1f)\ (c.2Epred_set_2EEMPTY \\
A.27a)) = (c.2Epred_set_2EEMPTY\ A.27a))) \wedge ((\forall V2f \in (A.27b^{A.27c}). \\
(\forall V3x \in A.27c.(\forall V4s \in (2^{A.27c}).((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27c\ A.27b)\ V0op)\ V2f)\ (ap\ (ap\ (c.2Epred_set_2EINSERT\ A.27c) \\
V3x)\ V4s)) = (ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ (2^{A.27c}))\ (ap\ (ap\ (c.2Emin_2E_3D \\
A.27b)\ (ap\ V2f\ V3x))\ (ap\ (c.2Eiterate_2Eneutral\ A.27b)\ V0op)))) \\
(ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27c\ A.27b)\ V0op)\ V2f)\ V4s))) \\
(ap\ (ap\ (c.2Epred_set_2EINSERT\ A.27c)\ V3x)\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27c\ A.27b)\ V0op)\ V2f)\ V4s)))))) \wedge ((\forall V5f \in (A.27b^{A.27d}). \\
(\forall V6x \in A.27d.(\forall V7s \in (2^{A.27d}).((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27d\ A.27b)\ V0op)\ V5f)\ (ap\ (ap\ (c.2Epred_set_2EDELETE\ A.27d) \\
V7s)\ V6x)) = (ap\ (ap\ (c.2Epred_set_2EDELETE\ A.27d)\ (ap\ (ap\ (ap\ (\\
c.2Eiterate_2Esupport\ A.27d\ A.27b)\ V0op)\ V5f)\ V7s)))\ V6x)))))) \wedge \\
((\forall V8f \in (A.27b^{A.27e}).(\forall V9s \in (2^{A.27e}).(\forall V10t \in \\
(2^{A.27e}).((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27e\ A.27b)\ V0op) \\
V8f)\ (ap\ (ap\ (c.2Epred_set_2EUNION\ A.27e)\ V9s)\ V10t)) = (ap\ (ap \\
(c.2Epred_set_2EUNION\ A.27e)\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27e\ A.27b)\ V0op)\ V8f)\ V9s))\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27e\ A.27b)\ V0op)\ V8f)\ V10t)))))) \wedge ((\forall V11f \in (A.27b^{A.27f}). \\
(\forall V12s \in (2^{A.27f}).(\forall V13t \in (2^{A.27f}).((ap\ (ap\ (ap \\
(c.2Eiterate_2Esupport\ A.27f\ A.27b)\ V0op)\ V11f)\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
A.27f)\ V12s)\ V13t)) = (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27f)\ (ap \\
(ap\ (ap\ (c.2Eiterate_2Esupport\ A.27f\ A.27b)\ V0op)\ V11f)\ V12s)) \\
(ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27f\ A.27b)\ V0op)\ V11f)\ V13t)))))) \wedge \\
((\forall V14f \in (A.27b^{A.27g}).(\forall V15s \in (2^{A.27g}).(\forall V16t \in \\
(2^{A.27g}).((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27g\ A.27b)\ V0op) \\
V14f)\ (ap\ (ap\ (c.2Epred_set_2EDIFF\ A.27g)\ V15s)\ V16t)) = (ap\ (ap \\
(c.2Epred_set_2EDIFF\ A.27g)\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27g\ A.27b)\ V0op)\ V14f)\ V15s))\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27g\ A.27b)\ V0op)\ V14f)\ V16t)))))) \wedge ((\forall V17f \in (A.27i^{A.27h}). \\
(\forall V18g \in (A.27b^{A.27i}).(\forall V19s \in (2^{A.27h}).((ap\ (ap \\
(ap\ (c.2Eiterate_2Esupport\ A.27i\ A.27b)\ V0op)\ V18g)\ (ap\ (ap\ (c.2Epred_set_2EIMAGE \\
A.27h\ A.27i)\ V17f)\ V19s)) = (ap\ (ap\ (c.2Epred_set_2EIMAGE\ A.27h \\
A.27i)\ V17f)\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27h\ A.27b)\ V0op) \\
(ap\ (ap\ (c.2Ecombin_2Eo\ A.27h\ A.27b\ A.27i)\ V18g)\ V17f))\ V19s)))))))))
\end{aligned}$$

(14)

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0op \in ((A_27a^{A_27a})^{A_27a}).(\forall V1s \in (2^{A_27b}).(\\
& \quad \forall V2f \in (A_27a^{A_27b}).(\forall V3a \in A_27b.((ap\ (ap\ (ap\ (c_2Eiterate_2Esupport \\
& \quad A_27b\ A_27a)\ V0op)\ (\lambda V4x \in A_27b.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& \quad A_27a)\ (ap\ (ap\ (c_2Emin_2E_3D\ A_27b)\ V4x)\ V3a))\ (ap\ V2f\ V4x))\ (ap \\
& \quad (c_2Eiterate_2Eneutral\ A_27a)\ V0op))))\ V1s) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& \quad (2^{A_27b}))\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V3a)\ V1s))\ (ap\ (ap\ (ap\ (\\
& \quad c_2Eiterate_2Esupport\ A_27b\ A_27a)\ V0op)\ V2f)\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad A_27b)\ V3a)\ (c_2Epred_set_2EEMPTY\ A_27b))))))\ (c_2Epred_set_2EEMPTY \\
& \quad A_27b))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0op \in ((A_27a^{A_27a})^{A_27a}).(\forall V1f \in (A_27a^{A_27b}). \\
& \quad (\forall V2s \in (2^{A_27b}).((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b \\
& \quad A_27a)\ V0op)\ (ap\ (ap\ (ap\ (c_2Eiterate_2Esupport\ A_27b\ A_27a)\ V0op) \\
& \quad V1f)\ V2s))\ V1f) = (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b\ A_27a) \\
& \quad V0op)\ V2s)\ V1f))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0op \in ((A_27a^{A_27a})^{A_27a}).((p\ (ap\ (c_2Eiterate_2Emonoidal \\
& \quad A_27a)\ V0op)) \Rightarrow ((\forall V1f \in (A_27a^{A_27b}).((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate \\
& \quad A_27b\ A_27a)\ V0op)\ (c_2Epred_set_2EEMPTY\ A_27b))\ V1f) = (ap\ (c_2Eiterate_2Eneutral \\
& \quad A_27a)\ V0op))) \wedge (\forall V2f \in (A_27a^{A_27b}).(\forall V3x \in A_27b. \\
& \quad (\forall V4s \in (2^{A_27b}).((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b) \\
& \quad V4s)) \Rightarrow ((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b\ A_27a)\ V0op)\ (\\
& \quad ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27b)\ V3x)\ V4s))\ V2f) = (ap\ (ap\ (\\
& \quad ap\ (c_2Ebool_2ECOND\ A_27a)\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V3x)\ V4s)) \\
& \quad (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b\ A_27a)\ V0op)\ V4s)\ V2f)) \\
& \quad (ap\ (ap\ V0op\ (ap\ V2f\ V3x))\ (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b \\
& \quad A_27a)\ V0op)\ V4s)\ V2f))))))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0op \in ((A_27b^{A_27b})^{A_27b}).((p\ (ap\ (c_2Eiterate_2Emonoidal \\
& \quad A_27b)\ V0op)) \Rightarrow (\forall V1f \in (A_27b^{A_27a}).(\forall V2x \in A_27a. \\
& \quad ((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27a\ A_27b)\ V0op)\ (ap\ (ap\ (\\
& \quad c_2Epred_set_2EINSERT\ A_27a)\ V2x)\ (c_2Epred_set_2EEMPTY\ A_27a))) \\
& \quad V1f) = (ap\ V1f\ V2x))))))
\end{aligned} \tag{18}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0op \in ((A_27a^{A_27a})^{A_27a}).((p\ (ap\ (c_2Eiterate_2Emonoidal \\ & A_27a)\ V0op)) \Rightarrow (\forall V1f \in (A_27a^{A_27b}).(\forall V2a \in A_27b. \\ & (\forall V3s \in (2^{A_27b}).((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b \\ & A_27a)\ V0op)\ V3s)\ (\lambda V4x \in A_27b.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\ & A_27a)\ (ap\ (ap\ (c_2Emin_2E_3D\ A_27b)\ V4x)\ V2a))\ (ap\ V1f\ V4x))\ (ap \\ & (c_2Eiterate_2Eneutral\ A_27a)\ V0op)))))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\ & A_27a)\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V2a)\ V3s))\ (ap\ V1f\ V2a))\ (ap \\ & (c_2Eiterate_2Eneutral\ A_27a)\ V0op)))))) \end{aligned}$$