

thm_2Eiterate_2EITERATE__DIFF__GEN (TM-FWK6KMRKh5vvUJvUtW9kcQw5AYEQyykDb)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 8 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_2F_5C$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 11 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 12 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 14 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 16 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2E$

Definition 17 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 18 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (ap$

Definition 19 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 20 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Emin$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 22 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 23 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2)$

Definition 24 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$

Definition 25 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 26 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 27 We define $c_2Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (ap c_2E$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{5}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{8}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{9}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t))))))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow (\\
& \forall V0op \in ((A.27b^{A.27b})^{A.27b}).((\forall V1f \in (A.27b^{A.27a}). \\
((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27a\ A.27b)\ V0op)\ V1f)\ (c.2Epred_set_2EEMPTY \\
A.27a)) = (c.2Epred_set_2EEMPTY\ A.27a))) \wedge ((\forall V2f \in (A.27b^{A.27c}). \\
(\forall V3x \in A.27c.(\forall V4s \in (2^{A.27c}).((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27c\ A.27b)\ V0op)\ V2f)\ (ap\ (ap\ (c.2Epred_set_2EINSERT\ A.27c) \\
V3x)\ V4s)) = (ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ (2^{A.27c}))\ (ap\ (ap\ (c.2Emin_2E_3D \\
A.27b)\ (ap\ V2f\ V3x))\ (ap\ (c.2Eiterate_2Eneutral\ A.27b)\ V0op)))) \\
(ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27c\ A.27b)\ V0op)\ V2f)\ V4s))) \\
(ap\ (ap\ (c.2Epred_set_2EINSERT\ A.27c)\ V3x)\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27c\ A.27b)\ V0op)\ V2f)\ V4s)))))) \wedge ((\forall V5f \in (A.27b^{A.27d}). \\
(\forall V6x \in A.27d.(\forall V7s \in (2^{A.27d}).((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27d\ A.27b)\ V0op)\ V5f)\ (ap\ (ap\ (c.2Epred_set_2EDELETE\ A.27d) \\
V7s)\ V6x)) = (ap\ (ap\ (c.2Epred_set_2EDELETE\ A.27d)\ (ap\ (ap\ (ap\ (\\
c.2Eiterate_2Esupport\ A.27d\ A.27b)\ V0op)\ V5f)\ V7s)))\ V6x)))))) \wedge \\
((\forall V8f \in (A.27b^{A.27e}).(\forall V9s \in (2^{A.27e}).(\forall V10t \in \\
(2^{A.27e}).((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27e\ A.27b)\ V0op) \\
V8f)\ (ap\ (ap\ (c.2Epred_set_2EUNION\ A.27e)\ V9s)\ V10t)) = (ap\ (ap \\
(c.2Epred_set_2EUNION\ A.27e)\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27e\ A.27b)\ V0op)\ V8f)\ V9s))\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27e\ A.27b)\ V0op)\ V8f)\ V10t)))))) \wedge ((\forall V11f \in (A.27b^{A.27f}). \\
(\forall V12s \in (2^{A.27f}).(\forall V13t \in (2^{A.27f}).((ap\ (ap\ (ap \\
(c.2Eiterate_2Esupport\ A.27f\ A.27b)\ V0op)\ V11f)\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
A.27f)\ V12s)\ V13t)) = (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27f)\ (ap \\
(ap\ (ap\ (c.2Eiterate_2Esupport\ A.27f\ A.27b)\ V0op)\ V11f)\ V12s)) \\
(ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27f\ A.27b)\ V0op)\ V11f)\ V13t)))))) \wedge \\
((\forall V14f \in (A.27b^{A.27g}).(\forall V15s \in (2^{A.27g}).(\forall V16t \in \\
(2^{A.27g}).((ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27g\ A.27b)\ V0op) \\
V14f)\ (ap\ (ap\ (c.2Epred_set_2EDIFF\ A.27g)\ V15s)\ V16t)) = (ap\ (ap \\
(c.2Epred_set_2EDIFF\ A.27g)\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27g\ A.27b)\ V0op)\ V14f)\ V15s))\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport \\
A.27g\ A.27b)\ V0op)\ V14f)\ V16t)))))) \wedge ((\forall V17f \in (A.27i^{A.27h}). \\
(\forall V18g \in (A.27b^{A.27i}).(\forall V19s \in (2^{A.27h}).((ap\ (ap \\
(ap\ (c.2Eiterate_2Esupport\ A.27i\ A.27b)\ V0op)\ V18g)\ (ap\ (ap\ (c.2Epred_set_2EIMAGE \\
A.27h\ A.27i)\ V17f)\ V19s)) = (ap\ (ap\ (c.2Epred_set_2EIMAGE\ A.27h \\
A.27i)\ V17f)\ (ap\ (ap\ (ap\ (c.2Eiterate_2Esupport\ A.27h\ A.27b)\ V0op) \\
(ap\ (ap\ (c.2Ecombin_2Eo\ A.27h\ A.27b\ A.27i)\ V18g)\ V17f))\ V19s)))))))))
\end{aligned}$$

(13)

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0op \in ((A_27a^{A_27a})^{A_27a}). (\forall V1f \in (A_27a^{A_27b}). \\
& \quad (\forall V2s \in (2^{A_27b}). ((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b \\
& \quad A_27a)\ V0op)\ (ap\ (ap\ (ap\ (c_2Eiterate_2Esupport\ A_27b\ A_27a)\ V0op) \\
& \quad V1f)\ V2s))\ V1f) = (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b\ A_27a) \\
& \quad V0op)\ V2s)\ V1f))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0op \in ((A_27a^{A_27a})^{A_27a}). ((p\ (ap\ (c_2Eiterate_2Emonoidal \\
& \quad A_27a)\ V0op)) \Rightarrow (\forall V1f \in (A_27a^{A_27b}). (\forall V2s \in (2^{A_27b}). \\
& \quad (\forall V3t \in (2^{A_27b}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b) \\
& \quad V2s)) \wedge (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27b)\ V3t)\ V2s))) \Rightarrow (\\
& \quad (ap\ (ap\ V0op\ (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b\ A_27a)\ V0op) \\
& \quad (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27b)\ V2s)\ V3t))\ V1f))\ (ap\ (ap\ (ap \\
& \quad (c_2Eiterate_2Eiterate\ A_27b\ A_27a)\ V0op)\ V3t)\ V1f)) = (ap\ (ap\ (\\
& \quad ap\ (c_2Eiterate_2Eiterate\ A_27b\ A_27a)\ V0op)\ V2s)\ V1f))))))
\end{aligned} \tag{15}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0op \in ((A_27b^{A_27b})^{A_27b}). ((p\ (ap\ (c_2Eiterate_2Emonoidal \\
& \quad A_27b)\ V0op)) \Rightarrow (\forall V1f \in (A_27b^{A_27a}). (\forall V2s \in (2^{A_27a}). \\
& \quad (\forall V3t \in (2^{A_27a}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\
& \quad (ap\ (ap\ (ap\ (c_2Eiterate_2Esupport\ A_27a\ A_27b)\ V0op)\ V1f)\ V2s))) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ (ap\ (ap\ (ap\ (c_2Eiterate_2Esupport \\
& \quad A_27a\ A_27b)\ V0op)\ V1f)\ V3t))\ (ap\ (ap\ (ap\ (c_2Eiterate_2Esupport \\
& \quad A_27a\ A_27b)\ V0op)\ V1f)\ V2s)))) \Rightarrow ((ap\ (ap\ V0op\ (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate \\
& \quad A_27a\ A_27b)\ V0op)\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ V2s)\ V3t)) \\
& \quad V1f))\ (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27a\ A_27b)\ V0op)\ V3t) \\
& \quad V1f)) = (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27a\ A_27b)\ V0op)\ V2s) \\
& \quad V1f))))))
\end{aligned}$$