

# thm\_2Eiterate\_2EITERATE\_\_INJECTION (TMX1zpwsEfE842TAPbvrPvNcD6ypiMyigmY)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).)(ap V1f V0x))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).)(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \tag{3}$$

**Definition 9** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2$

**Definition 10** We define `c_2Ebool_2EF` to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define `c_2Epred__set_2EEMPTY` to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 12** We define `c_2Epred__set_2EFINITE` to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_21\ 2)\ s)$ .

**Definition 13** We define `c_2Ecombin_2Eo` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27c^{A\_27b}).\lambda V2h \in (A\_27a^{A\_27b \times A\_27c}).(ap\ (c\_2Ebool\_2EF)\ f\ g\ h)$ .

**Definition 14** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A.\lambda y.y \in A).\mathbf{if}\ (ap\ P\ x)\ \mathbf{then}\ y)\ \mathbf{else}\ (the\ (\lambda x.x \in A.\lambda y.y \in A).\mathbf{if}\ (ap\ P\ y)\ \mathbf{then}\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 15** We define `c_2Eiterate_2Eneutral` to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_2Emin\_2E\_40)\ op)$ .

**Definition 16** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$ .

**Definition 17** We define `c_2Eiterate_2Esupport` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V1p \in (A\_27b^{A\_27a}).(ap\ (c\_2Emin\_2E\_40)\ op\ p)$ .

**Definition 18** We define `c_2Ebool_2ECOND` to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ (c\_2Ebool\_2EF)\ t1\ t2))))$ .

**Definition 19** We define `c_2Eiterate_2EITSET` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V1g \in (A\_27b^{A\_27a}).(ap\ (c\_2Emin\_2E\_40)\ f\ g)$ .

**Definition 20** We define `c_2Eiterate_2Eiterate` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V1p \in (A\_27b^{A\_27a}).(ap\ (c\_2Emin\_2E\_40)\ op\ p)$ .

**Definition 21** We define `c_2Ebool_2E_3F` to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40)\ P)))$ .

**Definition 22** We define `c_2Ebool_2E_3F_21` to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ c\_2Ebool\_2E\_2F\ 5C)\ P)))$ .

**Definition 23** We define `c_2Eiterate_2Emonoidal` to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (ap\ c\_2Emin\_2E\_40)\ op)$ .

**Definition 24** We define `c_2Epred__set_2ESUBSET` to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2EF)\ s\ t))$ .

**Definition 25** We define `c_2Epred__set_2EIMAGE` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (A\_27b^{A\_27b}).(ap\ (c\_2Emin\_2E\_40)\ f\ s))$ .

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (14)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\exists V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.(((\exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \vee (p \ V1Q)) \Leftrightarrow (\exists V3x \in A.27a.((p \ (ap \ V0P \ V3x)) \vee (p \ V1Q)))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p \ V0P) \vee (\exists V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V3x)))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \ V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((\neg((p \ V0A) \Rightarrow (p \ V1B))) \Leftrightarrow ((p \ V0A) \wedge (\neg(p \ V1B)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((p \ (ap \\ (c.2Ebool.2E.3F.21 \ A.27a) \ (\lambda V1x \in A.27a.(ap \ V0P \ V1x)))) \Leftrightarrow (( \\ \exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \wedge (\forall V3x \in A.27a.(\forall V4y \in \\ A.27a.(((p \ (ap \ V0P \ V3x)) \wedge (p \ (ap \ V0P \ V4y))) \Rightarrow (V3x = V4y)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow ( \\ \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p \ (ap \ (ap \ V0P \ V1x) \ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).( \\ \forall V4x \in A.27a.(p \ (ap \ (ap \ V0P \ V4x) \ (ap \ V3f \ V4x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1f \in \\ (A.27a^{A.27a}).(((p \ (ap \ (c.2Epred\_set.2EFINITE \ A.27a) \ V0s)) \wedge \\ (p \ (ap \ (ap \ (c.2Epred\_set.2ESUBSET \ A.27a) \ (ap \ (ap \ (c.2Epred\_set.2EIMAGE \\ A.27a \ A.27a) \ V1f) \ V0s))) \Rightarrow ((\forall V2y \in A.27a.((p \ (ap \ (ap \\ (c.2Ebool.2EIN \ A.27a) \ V2y) \ V0s))) \Rightarrow (\exists V3x \in A.27a.((p \ (ap \ ( \\ ap \ (c.2Ebool.2EIN \ A.27a) \ V3x) \ V0s)) \wedge ((ap \ V1f \ V3x) = V2y)))))) \Leftrightarrow (\forall V4x \in \\ A.27a.(\forall V5y \in A.27a.(((p \ (ap \ (ap \ (c.2Ebool.2EIN \ A.27a) \ V4x) \\ V0s)) \wedge ((p \ (ap \ (ap \ (c.2Ebool.2EIN \ A.27a) \ V5y) \ V0s)) \wedge ((ap \ V1f \ V4x) = \\ (ap \ V1f \ V5y)))))) \Rightarrow (V4x = V5y)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow ( \\ \forall V0op \in ((A.27b^{A.27b})^{A.27a}).((p \ (ap \ (c.2Eiterate.2Emonoidal \\ A.27b) \ V0op)) \Rightarrow (\forall V1f \in (A.27b^{A.27a}).(\forall V2p \in (A.27a^{A.27a}). \\ (\forall V3s \in (2^{A.27a}).(((\forall V4x \in A.27a.((p \ (ap \ (ap \ (c.2Ebool.2EIN \\ A.27a) \ V4x) \ V3s))) \Rightarrow (p \ (ap \ (ap \ (c.2Ebool.2EIN \ A.27a) \ (ap \ V2p \ V4x)) \\ V3s)))))) \wedge (\forall V5y \in A.27a.((p \ (ap \ (ap \ (c.2Ebool.2EIN \ A.27a) \\ V5y) \ V3s))) \Rightarrow (p \ (ap \ (c.2Ebool.2E.3F.21 \ A.27a) \ (\lambda V6x \in A.27a. \\ (ap \ (ap \ c.2Ebool.2E.2F.5C \ (ap \ (ap \ (c.2Ebool.2EIN \ A.27a) \ V6x) \ V3s)) \\ (ap \ (ap \ (c.2Emin.2E.3D \ A.27a) \ (ap \ V2p \ V6x)) \ V5y)))))) \Rightarrow ((ap \ (ap \\ (ap \ (c.2Eiterate.2Eiterate \ A.27a \ A.27b) \ V0op) \ V3s) \ V1f) = (ap \ (ap \\ (ap \ (c.2Eiterate.2Eiterate \ A.27a \ A.27b) \ V0op) \ V3s) \ (ap \ (ap \ (c.2Ecombin.2Eo \\ A.27a \ A.27b \ A.27a) \ V1f) \ V2p)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0y \in A\_27b. (\forall V1s \in (2^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}). \\
& \quad ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& \quad A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s))))))
\end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& \quad (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& \quad ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q)))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (48)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0op \in ((A\_27b^{A\_27b})^{A\_27b}). ((p\ (ap\ (c\_2Eiterate\_2Emonoidal\ A\_27b)\ V0op)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}). (\forall V2p \in (A\_27a^{A\_27a}). \\ & (\forall V3s \in (2^{A\_27a}). (((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V3s)) \wedge ((\forall V4x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V4x)\ V3s)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ (ap\ V2p\ V4x))\ V3s)))))) \wedge (\forall V5x \in \\ & A\_27a. (\forall V6y \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V5x)\ V3s)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V6y)\ V3s)) \wedge ((ap\ V2p\ V5x) = \\ & (ap\ V2p\ V6y)))))) \Rightarrow (V5x = V6y)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ A\_27b)\ V0op)\ V3s)\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b\ A\_27a)\ V1f)\ V2p)) = (ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ A\_27b)\ V0op)\ V3s)\ V1f)))))) \end{aligned}$$