

thm\_2Eiterate\_2EITERATE\_\_RELATED  
(TMQTTphnnqDevJD-  
KXVwhkissvCW4bHbTPro)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 8** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

**Definition 9** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap (c\_2Emin\_2E\_3D$

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 11** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
(3)

**Definition 13** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 14** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 15** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2E$

**Definition 16** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF).$

**Definition 17** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 2)$

**Definition 18** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

**Definition 19** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 20** We define  $c\_2Eiterate\_2Emonoidal$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap (ap c\_2E$

Assume the following.

$$True$$
(4)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))$$
(5)

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t)))$$
(6)

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p V0t) \Leftrightarrow (p V0t)))$$
(7)

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))$$
(8)

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))$$
(9)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (14)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27)))))) \quad (16)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0op \in ((A\_27a^{A\_27a})^{A\_27a}).((p\ (ap\ (c\_2Eiterate\_2Emonoidal \\
& \quad A\_27a)\ V0op)) \Rightarrow ((\forall V1f \in (A\_27a^{A\_27b}).((ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate \\
& \quad A\_27b\ A\_27a)\ V0op)\ (c\_2Epred\_set\_2EEMPTY\ A\_27b))\ V1f) = (ap\ (c\_2Eiterate\_2Eneutral \\
& \quad A\_27a)\ V0op))) \wedge (\forall V2f \in (A\_27a^{A\_27b}).(\forall V3x \in A\_27b. \\
& \quad (\forall V4s \in (2^{A\_27b}).((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27b) \\
& \quad V4s)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate\ A\_27b\ A\_27a)\ V0op)\ ( \\
& \quad ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27b)\ V3x)\ V4s))\ V2f) = (ap\ (ap\ ( \\
& \quad ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V3x)\ V4s)) \\
& \quad (ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate\ A\_27b\ A\_27a)\ V0op)\ V4s)\ V2f)) \\
& \quad (ap\ (ap\ V0op\ (ap\ V2f\ V3x))\ (ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate\ A\_27b \\
& \quad A\_27a)\ V0op)\ V4s)\ V2f))))))))) \\
& \hspace{15em} (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& \quad A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\
& \quad V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\
& \quad V1y) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V2s)))))) \\
& \hspace{15em} (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}).(( \\
& \quad (p\ (ap\ V0P\ (c\_2Epred\_set\_2EEMPTY\ A\_27a))) \wedge (\forall V1s \in (2^{A\_27a}). \\
& \quad ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\
& \quad (\forall V2e \in A\_27a.((\neg(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2e)\ V1s))) \Rightarrow \\
& \quad (p\ (ap\ V0P\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V2e)\ V1s)))))) \Rightarrow \\
& \quad (\forall V3s \in (2^{A\_27a}).((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a) \\
& \quad V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \\
& \hspace{15em} (19)
\end{aligned}$$

### Theorem 1

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0op \in ((A\_27b^{A\_27b})^{A\_27b}).((p\ (ap\ (c\_2Eiterate\_2Emonoidal \\
& \quad A\_27b)\ V0op)) \Rightarrow (\forall V1R \in ((2^{A\_27b})^{A\_27b}).(((p\ (ap\ (ap\ V1R \\
& \quad (ap\ (c\_2Eiterate\_2Eneutral\ A\_27b)\ V0op))\ (ap\ (c\_2Eiterate\_2Eneutral \\
& \quad A\_27b)\ V0op))) \wedge (\forall V2x1 \in A\_27b.(\forall V3y1 \in A\_27b.(\forall V4x2 \in \\
& \quad A\_27b.(\forall V5y2 \in A\_27b.(((p\ (ap\ (ap\ V1R\ V2x1)\ V4x2)) \wedge (p\ (ap \\
& \quad (ap\ V1R\ V3y1)\ V5y2))) \Rightarrow (p\ (ap\ (ap\ V1R\ (ap\ (ap\ V0op\ V2x1)\ V3y1))\ (ap\ ( \\
& \quad ap\ V0op\ V4x2)\ V5y2)))))) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}).(\forall V7g \in \\
& \quad (A\_27b^{A\_27a}).(\forall V8s \in (2^{A\_27a}).(((p\ (ap\ (c\_2Epred\_set\_2EFINITE \\
& \quad A\_27a)\ V8s)) \wedge (\forall V9x \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\
& \quad V9x)\ V8s)) \Rightarrow (p\ (ap\ (ap\ V1R\ (ap\ V6f\ V9x))\ (ap\ V7g\ V9x)))))) \Rightarrow (p\ (ap\ (ap \\
& \quad V1R\ (ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ A\_27b)\ V0op)\ V8s)\ V6f)) \\
& \quad (ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ A\_27b)\ V0op)\ V8s)\ V7g))))))))) \\
& \hspace{15em}
\end{aligned}$$