thm_2Eiterate_2EITERATE__UNION__GEN (TMFD-cxhC2tn4ziaTLmT5LoLn7NTTVthwNKC)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 11 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 12 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 13 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 14 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 15 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 16 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 17 *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Definition 18 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A$.**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 19 *We define* c_2Eiterate_2Eneutral *to be* $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ ($c_2Emin

Definition 20 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 21 *We define* c_2Eiterate_2Esupport *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 22 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 23 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ ($c_2Ebool_2E_21 $(2$

Definition 24 *We define* c_2Eiterate_2EITSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

Definition 25 *We define* c_2Eiterate_2Eiterate *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 26 *We define* c_2Epred__set_2EDISJOINT *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$

Definition 27 *We define* c_2Eiterate_2Emonoidal *to be* $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (ap$ c_2

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{11}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow \forall A\_27g.nonempty\ A\_27g \Rightarrow \forall A\_27h.nonempty\ A\_27h \Rightarrow \forall A\_27i.nonempty\ A\_27i \Rightarrow (\forall V0op \in ((A\_27b^{A\_27b})^{A\_27b}).((\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27a\ A\_27b)\ V0op)\ V1f)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \wedge ((\forall V2f \in (A\_27b^{A\_27c}).(\forall V3x \in A\_27c.(\forall V4s \in (2^{A\_27c}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27c\ A\_27b)\ V0op)\ V2f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27c)\ V3x)\ V4s)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (2^{A\_27c}))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27b)\ (ap\ V2f\ V3x))\ (ap\ (\mathsf{c\_2Eiterate\_2Eneutral}\ A\_27b)\ V0op)))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27c\ A\_27b)\ V0op)\ V2f)\ V4s))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27c)\ V3x)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27c\ A\_27b)\ V0op)\ V2f)\ V4s)))))))) \wedge ((\forall V5f \in (A\_27b^{A\_27d}).(\forall V6x \in A\_27d.(\forall V7s \in (2^{A\_27d}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27d\ A\_27b)\ V0op)\ V5f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27d)\ V7s)\ V6x)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27d)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27d\ A\_27b)\ V0op)\ V5f)\ V7s))\ V6x))))) \wedge ((\forall V8f \in (A\_27b^{A\_27e}).(\forall V9s \in (2^{A\_27e}).(\forall V10t \in (2^{A\_27e}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27e\ A\_27b)\ V0op)\ V8f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27e)\ V9s)\ V10t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27e)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27e\ A\_27b)\ V0op)\ V8f)\ V9s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27e\ A\_27b)\ V0op)\ V8f)\ V10t)))))) \wedge ((\forall V11f \in (A\_27b^{A\_27f}).(\forall V12s \in (2^{A\_27f}).(\forall V13t \in (2^{A\_27f}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27f\ A\_27b)\ V0op)\ V11f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27f)\ V12s)\ V13t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27f)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27f\ A\_27b)\ V0op)\ V11f)\ V12s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27f\ A\_27b)\ V0op)\ V11f)\ V13t)))))) \wedge ((\forall V14f \in (A\_27b^{A\_27g}).(\forall V15s \in (2^{A\_27g}).(\forall V16t \in (2^{A\_27g}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27g\ A\_27b)\ V0op)\ V14f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27g)\ V15s)\ V16t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27g)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27g\ A\_27b)\ V0op)\ V14f)\ V15s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27g\ A\_27b)\ V0op)\ V14f)\ V16t)))))) \wedge (\forall V17f \in (A\_27i^{A\_27h}).(\forall V18g \in (A\_27b^{A\_27i}).(\forall V19s \in (2^{A\_27h}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27i\ A\_27b)\ V0op)\ V18g)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27h\ A\_27i)\ V17f)\ V19s)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27h\ A\_27i)\ V17f)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27h\ A\_27b)\ V0op)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27h\ A\_27b\ A\_27i)\ V18g)\ V17f))\ V19s)))))))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(\forall V1f \in (A\_27a^{A\_27b}). (\forall V2s \in (2^{A\_27b}).((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27b\ A\_27a)\ V0op)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27b\ A\_27a)\ V0op)\ V1f)\ V2s))\ V1f) = (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27b\ A\_27a)\ V0op)\ V2s)\ V1f)))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0op \in ((A\_27a^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Eiterate\_2Emonoidal}\ A\_27a)\ V0op)) \Rightarrow (\forall V1f \in (A\_27a^{A\_27b}).(\forall V2s \in (2^{A\_27b}). (\forall V3t \in (2^{A\_27b}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ V2s)) \wedge ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ V3t)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDISJOINT}\ A\_27b)\ V2s)\ V3t)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27b\ A\_27a)\ V0op)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27b)\ V2s)\ V3t))\ V1f) = (ap\ (ap\ V0op\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27b\ A\_27a)\ V0op)\ V2s)\ V1f))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27b\ A\_27a)\ V0op)\ V3t)\ V1f)))))))))) \tag{15}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0op \in ((A\_27b^{A\_27b})^{A\_27b}).((p\ (ap\ (\mathsf{c\_2Eiterate\_2Emonoidal}\ A\_27b)\ V0op)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}).(\forall V2s \in (2^{A\_27a}). (\forall V3t \in (2^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27a\ A\_27b)\ V0op)\ V1f)\ V2s))) \wedge ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27a\ A\_27b)\ V0op)\ V1f)\ V3t))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDISJOINT}\ A\_27a)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27a\ A\_27b)\ V0op)\ V1f)\ V2s))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Esupport}\ A\_27a\ A\_27b)\ V0op)\ V1f)\ V3t))))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27b)\ V0op)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V2s)\ V3t))\ V1f) = (ap\ (ap\ V0op\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27b)\ V0op)\ V2s)\ V1f))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27b)\ V0op)\ V3t)\ V1f)))))))))$$