

thm_2Eiterate_2ENSUM__BIJECTION (TMJ8C8nrJznA4jCVBHfatnv1T9NjhFMvT1T)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 12 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 13 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Emin$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (3)$$

Definition 15 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27a}).\lambda V$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in$

Definition 18 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2E$

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 20 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21\ 2) (\lambda V$

Definition 21 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$

Definition 22 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27a}).\lambda V$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Definition 23 We define $c_2Eiterate_2Eenum$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate\ A_27a\ ty_2Enum_2Enum) (\lambda V$

Definition 24 We define $c_2Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (ap c_2E$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \quad (7) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0op \in ((A_27b^{A_27b})^{A_27b}).((p\ (ap\ (c_2Eiterate_2Emonoidal \\
& A_27b)\ V0op)) \Rightarrow (\forall V1f \in (A_27b^{A_27a}).(\forall V2p \in (A_27a^{A_27a}). \\
& (\forall V3s \in (2^{A_27a}).((\forall V4x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V4x)\ V3s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ (ap\ V2p\ V4x)) \\
& V3s)))) \wedge (\forall V5y \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V5y)\ V3s)) \Rightarrow (p\ (ap\ (c_2Ebool_2E_3F_21\ A_27a)\ (\lambda V6x \in A_27a.(\\
& ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V6x)\ V3s)) \\
& (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ (ap\ V2p\ V6x))\ V5y)))))) \Rightarrow ((ap\ (ap \\
& (ap\ (c_2Eiterate_2Eiterate\ A_27a\ A_27b)\ V0op)\ V3s)\ V1f) = (ap\ (ap \\
& (ap\ (c_2Eiterate_2Eiterate\ A_27a\ A_27b)\ V0op)\ V3s)\ (ap\ (ap\ (c_2Ecombin_2Eo \\
& A_27a\ A_27b\ A_27a)\ V1f)\ V2p)))))))))
\end{aligned} \tag{8}$$

Assume the following.

$$(p\ (ap\ (c_2Eiterate_2Emonoidal\ ty_2Enum_2Enum)\ c_2Earithmetic_2E_2B)) \tag{9}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Enum_2Enum^{A_27a}). \\
& (\forall V1p \in (A_27a^{A_27a}).(\forall V2s \in (2^{A_27a}).((\forall V3x \in \\
& A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V2s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ (ap\ V1p\ V3x))\ V2s)))) \wedge (\forall V4y \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V4y)\ V2s)) \Rightarrow (p\ (ap\ (c_2Ebool_2E_3F_21\ A_27a)\ (\lambda V5x \in A_27a. \\
& (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V5x)\ V2s)) \\
& (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ (ap\ V1p\ V5x))\ V4y)))))) \Rightarrow ((ap\ (ap \\
& (c_2Eiterate_2Eenum\ A_27a)\ V2s)\ V0f) = (ap\ (ap\ (c_2Eiterate_2Eenum \\
& A_27a)\ V2s)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ ty_2Enum_2Enum\ A_27a) \\
& V0f)\ V1p)))))))))
\end{aligned}$$