

thm_2Eiterate_2ENSUM__CLAUSES (TMU-jrAAzwyYEtS5R2wdrXC3jTwZWvPMjt61)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in (2^{V0P}).(ap (c_2Ebool_2E_3D (2^{V1P})))))))$

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 = V1t2))))))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 = V1t2))))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod \\ & \quad A0 A1) \end{aligned} \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ & \quad A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2Eprod A_27a A_27b) (inj_o (V0x = V1y)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & \quad A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a A_27b)^{A_27b}}) \end{aligned} \tag{3}$$

Definition 9 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}).(ap (c_2Epred_set_2EGSPEC A_27a V0x) (inj_o (V1s = V0x)))$

Definition 10 We define c_Ebool_2EF to be $(ap(c_Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2EF)$.

Definition 12 We define $c_Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap(c_Ebool_2E_21\ (2^{A_27a}))\ s)$.

Definition 13 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\lambda x.x \in A \wedge P(x) = 1) \text{ else } 0$.

Definition 14 We define $c_Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap(c_Emin_2E_40\ (2^{A_27a}))\ op)$.

Definition 15 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2EF))$.

Definition 16 We define $c_Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V0f \in ((A_27a^{A_27a})^{A_27a}).V0f$.

Definition 17 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(V0t = t1 \wedge V2t2 = t1) \vee (V0t = t2 \wedge V2t2 = t1))))$.

Definition 18 We define $c_Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V0g \in ((A_27b^{A_27b})^{A_27a}).V0f = g$.

Definition 19 We define $c_Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).V0f = f$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (4)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Definition 20 We define $c_Eiterate_2Esum$ to be $\lambda A_27a : \iota.(ap(c_Eiterate_2Eiterate\ A_27a\ ty_2Enum_2Enum)\ V0t)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (6)$$

Let $c_2Enum_2EAABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EAABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (7)$$

Definition 21 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EAABS_num\ c_2Enum_2EZERO_REP)$.

Definition 22 We define $c_Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ (ap\ c_2Eiterate_2Eiterate\ A_27a\ V0op)\ 1)$.

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (10)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (11)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (12)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (14)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.(p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V0P V3x)) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x))))))) \quad (15)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p V0P) \wedge (p (ap V1Q V3x))))))) \quad (16)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.(p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V1P V3x)) \vee (p V0Q)))))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A))))))) \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ & \quad \forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1b \in 2.(\forall V2x \in A_{27a}. \\ & \quad (\forall V3y \in A_{27a}.((ap\ V0f\ (ap\ (ap\ (ap\ (c_{2E}bool_2ECOND\ A_{27a}) \\ & \quad V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_{2E}bool_2ECOND\ A_{27b})\ V1b)\ (ap\ V0f\ \\ & \quad V2x))\ (ap\ V0f\ V3y))))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ & \quad \forall V0op \in ((A_{27a}^{A_{27a}})^{A_{27a}}).((p\ (ap\ (c_{2E}iterate_2Emonoidal\ \\ & \quad A_{27a})\ V0op)) \Rightarrow ((\forall V1f \in (A_{27a}^{A_{27b}}).((ap\ (ap\ (ap\ (c_{2E}iterate_2Eiterate\ \\ & \quad A_{27b}\ A_{27a})\ V0op)\ (c_{2E}pred_set_2EEMPTY\ A_{27b}))\ V1f) = (ap\ (c_{2E}iterate_2Eneutral\ \\ & \quad A_{27a})\ V0op))) \wedge (\forall V2f \in (A_{27a}^{A_{27b}}).(\forall V3x \in A_{27b}. \\ & \quad (\forall V4s \in (2^{A_{27b}}).((p\ (ap\ (c_{2E}pred_set_2EFINITE\ A_{27b})\ \\ & \quad V4s)) \Rightarrow ((ap\ (ap\ (ap\ (c_{2E}iterate_2Eiterate\ A_{27b}\ A_{27a})\ V0op)\ \\ & \quad ap\ (ap\ (c_{2E}pred_set_2EINSERT\ A_{27b})\ V3x))\ V2f) = (ap\ (ap\ (\\ & \quad ap\ (c_{2E}bool_2ECOND\ A_{27a})\ (ap\ (ap\ (c_{2E}bool_2EIN\ A_{27b})\ V3x)\ V4s)) \\ & \quad (ap\ (ap\ (ap\ (c_{2E}iterate_2Eiterate\ A_{27b}\ A_{27a})\ V0op)\ V4s)\ V2f)) \\ & \quad (ap\ (ap\ V0op\ (ap\ V2f\ V3x))\ (ap\ (ap\ (ap\ (c_{2E}iterate_2Eiterate\ A_{27b}\ \\ & \quad A_{27a})\ V0op)\ V4s)\ V2f))))))))))) \end{aligned} \quad (20)$$

Assume the following.

$$((ap\ (c_{2E}iterate_2Eneutral\ ty_2Enum_2Enum)\ c_{2E}arithmetric_2E_2B) = \\ c_{2E}Enum_2E0) \quad (21)$$

Assume the following.

$$(p\ (ap\ (c_{2E}iterate_2Emonoidal\ ty_2Enum_2Enum)\ c_{2E}arithmetric_2E_2B)) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q) \vee (\neg(p V0p)))))))))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (\neg(p V2r)))) \wedge (((p V0p) \vee ((\neg(p V2r) \vee (\neg(p V1q)))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee ((\neg(p V2r) \wedge \\ & (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q) \vee (\neg(p V0p))))))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(\forall V3s \in \\ & 2.(((p V0p) \Leftrightarrow (p (ap (ap (ap (c_2Ebool_2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\ & (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r) \vee (\neg(p V1q)))) \wedge \\ & (((p V0p) \vee ((\neg(p V2r) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p)))))))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (38)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow (\\ & (\forall V0f \in (ty_2Enum_2Enum^{A_{.27a}}). ((ap (ap (c_2Eiterate_2Ensum \\ & A_{.27a}) (c_2Epred_set_2EEMPTY A_{.27a})) V0f) = c_2Enum_2E0)) \wedge (\\ & \forall V1x \in A_{.27b}. (\forall V2f \in (ty_2Enum_2Enum^{A_{.27b}}). (\forall V3s \in \\ & (2^{A_{.27b}}). ((p (ap (c_2Epred_set_2EFINITE A_{.27b}) V3s)) \Rightarrow ((ap \\ & (ap (c_2Eiterate_2Ensum A_{.27b}) (ap (ap (c_2Epred_set_2EINSERT \\ & A_{.27b}) V1x) V3s)) V2f) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\ & (ap (ap (c_2Ebool_2EIN A_{.27b}) V1x) V3s)) (ap (ap (c_2Eiterate_2Ensum \\ & A_{.27b}) V3s) V2f)) (ap (ap c_2Earithmetic_2E_2B (ap V2f V1x)) (ap \\ & (ap (c_2Eiterate_2Ensum A_{.27b}) V3s) V2f)))))))))) \end{aligned}$$