thm_2Eiterate_2ENSUM__CLAUSES_LEFT (TMZgHQo1mA3zcNW2M1BZ14oSu2x6yDA3esR)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2EF

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

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Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Epair\_2Eprod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$

$$(5)$$

Let $c\_2Epair\_2EABS\_prod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EABS\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$

$$(6)$$

Definition 14 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_set\_2EGSPEC : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_set\_2EGSPEC$$
$$A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})$$

$$(7)$$

Definition 15 *We define* c_2Eiterate_2E_2E_2E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 16 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 18 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 19 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF}).$

Definition 20 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (2$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(8)$$

Definition 21 *We define* c_2Eiterate_2Eneutral *to be* $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Emin}$

Definition 22 *We define* c_2Eiterate_2Esupport *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 23 *We define* c_2Eiterate_2EITSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

Definition 24 *We define* c_2Eiterate_2Eiterate *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 25 *We define* c_2Eiterate_2Ensum *to be* $\lambda A\_27a : \iota.(ap$ (c_2Eiterate_2Eiterate $A\_27a\ ty\_2Enum\_2$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \qquad (9)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \qquad (10)$$

Definition 26 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 27 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (11)$$

Definition 28 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 29 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(12)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(13)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(14)$$

Definition 30 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 31 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 32 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 33 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 34 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \land (((ap\ (\\ ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ V0m)\ V1n))) \land ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC} \\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))))$$
$$(15)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ V1n)\ V0m))))$$
$$(16)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ \forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p)))))$$
$$(17)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \land \\ (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \land \\ (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \land \\ (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m) \land (\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap \\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)) \\ V1n)) \land ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\ V0m)\ V1n))))))))))$$
$$(18)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ \forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))))))$$
$$(19)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(V0m = V1n)) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}}$$
$$V0m))\ V1n)) \lor (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}}$$
$$V1n))\ V0m)))))) \tag{20}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}}$$
$$\mathsf{c\_2Earithmetic\_2EZERO}))\ V0n))) \tag{21}$$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p}$$
$$V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg(p\ V0t)))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow$$
$$(p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land$$
$$(((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \lor (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \lor True) \Leftrightarrow True) \land$$
$$(((False \lor (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \lor False) \Leftrightarrow (p\ V0t)) \land (((p\ V0t) \lor$$
$$(p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Rightarrow True) \Leftrightarrow$$
$$True) \land (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \land (($$
$$(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))))) \tag{29}$$

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Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge \\ ((\neg False)\Leftrightarrow True))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow \\ True)) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow \\ (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(\\ p\ V0t)))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1) \\ V1t2) = V0t1)\wedge((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF}) \\ V0t1)\ V1t2) = V1t2)))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(\\ p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\Rightarrow(p\ V1B))\Leftrightarrow((\neg(p\ V0A))\vee \\ (p\ V1B))))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Leftrightarrow((p\ V0t)\Leftrightarrow False))) \tag{37}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow \\ ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\\ \forall V0b \in 2.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2g \in (A\_27b^{A\_27a}). \\ (\forall V3x \in A\_27a.((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (A\_27b^{A\_27a})) \\ V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ V0b)\ (ap \\ V1f\ V3x))\ (ap\ V2g\ V3x))))))) \tag{39}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a.$$
$$(\forall V3y \in A\_27a.((ap\ V0f\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ V1b)\ (ap\ V0f$$
$$V2x))\ (ap\ V0f\ V3y)))))))$$

$$(40)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in$$
$$2.(((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow$$
$$(((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))$$

$$(41)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.$$
$$(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.$$
$$(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge$$
$$((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)$$
$$V5y\_27)))))))))$$

$$(42)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}$$
$$V0m)\ V1n)))))$$

$$(43)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}$$
$$ty\_2Enum\_2Enum)\ V0m)\ (ap\ (ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO}))))\ V1n)) = (ap\ (ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}$$
$$V0m)\ V1n)))))$$

$$(44)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)$$
$$V2p)\ (ap\ (ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}\ V0m)\ V1n))) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ V2p)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V2p)\ V1n)))))))$$

$$(45)$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$(\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Eiterate\_2Ensum}$$
$$A\_27a)\ (\mathsf{c\_2Epred\_set\_2EEMPTY}\ A\_27a))\ V0f) = \mathsf{c\_2Enum\_2E0})) \wedge ($$
$$\forall V1x \in A\_27b.(\forall V2f \in (ty\_2Enum\_2Enum^{A\_27b}).(\forall V3s \in$$
$$(2^{A\_27b}).((p\ (ap\ (\mathsf{c\_2Epred\_set\_2EFINITE}\ A\_27b)\ V3s)) \Rightarrow ((ap$$
$$(ap\ (\mathsf{c\_2Eiterate\_2Ensum}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_set\_2EINSERT}$$
$$A\_27b)\ V1x)\ V3s))\ V2f) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)$$
$$(ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V1x)\ V3s))\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Ensum}$$
$$A\_27b)\ V3s)\ V2f))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ V2f\ V1x))\ (ap$$
$$(ap\ (\mathsf{c\_2Eiterate\_2Ensum}\ A\_27b)\ V3s)\ V2f)))))))))$$

$$(46)$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
c_2Enum_2E0$)\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)$ c_2Enum_2E0$) = V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap$ c_2Earithmetic_2ENUMERAL $V2n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V3m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Enumeral_2EiZ $(ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A$ c_2Enum_2E0$)\ V4n) = $ c_2Enum_2E0$))\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)$ c_2Enum_2E0$) = $ c_2Enum_2E0$))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
ap c_2Earithmetic_2ENUMERAL $V6n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V7m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
c_2Enum_2E0$)\ V8n) = $ c_2Enum_2E0$))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)$ c_2Enum_2E0$) = V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap$ c_2Earithmetic_2ENUMERAL $V10n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V11m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
c_2Enum_2E0$)\ (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
$V12n))) = $ c_2Enum_2E0$))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP$ c_2Enum_2E0$)\ (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Earithmetic_2EBIT2 $V13n))) = $ c_2Enum_2E0$))\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)$ c_2Enum_2E0$) =$
$(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO$))))\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap$ c_2Earithmetic_2ENUMERAL $V15n))$
$(ap$ c_2Earithmetic_2ENUMERAL $V16m)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap$ c_2Enum_2ESUC
c_2Enum_2E0$) = (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
c_2Earithmetic_2EZERO$)))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap$ c_2Enum_2ESUC $(ap$ c_2Earithmetic_2ENUMERAL $V17n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Enum_2ESUC $V17n))))\wedge(((ap$ c_2Eprim__rec_2EPRE c_2Enum_2E0$) =$
c_2Enum_2E0$)\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap$ c_2Eprim__rec_2EPRE
$(ap$ c_2Earithmetic_2ENUMERAL $V18n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Eprim__rec_2EPRE $V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap$ c_2Earithmetic_2ENUMERAL $V19n) = $ c_2Enum_2E0$)\Leftrightarrow(V19n = $ c_2Earithmetic_2EZERO$)))\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.((c_2Enum_2E0 = (ap$ c_2Earithmetic_2ENUMERAL
$V20n))\Leftrightarrow(V20n = $ c_2Earithmetic_2EZERO$)))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap$ c_2Earithmetic_2ENUMERAL
$V21n) = (ap$ c_2Earithmetic_2ENUMERAL $V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V23n)$ c_2Enum_2E0$))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Enum_2E0$)\ (ap$ c_2Earithmetic_2ENUMERAL
$V24n)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO$)$
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C $(ap$ c_2Earithmetic_2ENUMERAL
$V25n))\ (ap$ c_2Earithmetic_2ENUMERAL $V26m)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Earithmetic_2E_3E
c_2Enum_2E0$)\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Earithmetic_2E_3E $(ap$ c_2Earithmetic_2ENUMERAL
$V28n))$ c_2Enum_2E0$))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO$)$
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Earithmetic_2E_3E $(ap$ c_2Earithmetic_2ENUMERAL
$V29n))\ (ap$ c_2Earithmetic_2ENUMERAL $V30m)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Earithmetic_2E_3C_3D
c_2Enum_2E0$)\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Earithmetic_2E_3C_3D $(ap$ c_2Earithmetic_2ENUMERAL

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}~\mathsf{c\_2Earithmetic\_2EZERO})~V0n)) \Leftrightarrow$$
$$True) \wedge (((p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}~(ap~\mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))~\mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap~\mathsf{c\_2Earithmetic\_2EBIT2}~V0n))~\mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge$$
$$(((p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}~(ap~\mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))~(ap~\mathsf{c\_2Earithmetic\_2EBIT1}~V1m))) \Leftrightarrow (p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)~V1m))) \wedge (((p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}~(ap~\mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))~(ap~\mathsf{c\_2Earithmetic\_2EBIT2}~V1m))) \Leftrightarrow (p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)~V1m))) \wedge (((p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}~(ap~\mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))~(ap~\mathsf{c\_2Earithmetic\_2EBIT1}~V1m))) \Leftrightarrow (\neg(p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)~V0n)))) \wedge ((p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}~(ap~\mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))~(ap~\mathsf{c\_2Earithmetic\_2EBIT2}~V1m))) \Leftrightarrow (p~(ap~(ap~\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)~V1m)))))))))))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p~V0t))) \Leftrightarrow (p~V0t))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.((p~V0A) \Rightarrow ((\neg(p~V0A)) \Rightarrow False))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p~V0A) \vee (p~V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p~V0A) \Rightarrow False) \Rightarrow ((\neg(p~V1B)) \Rightarrow False))))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p~V0A)) \vee (p~V1B))) \Rightarrow False) \Leftrightarrow$$
$$((p~V0A) \Rightarrow ((\neg(p~V1B)) \Rightarrow False))))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p~V0A)) \Rightarrow False) \Rightarrow (((p~V0A) \Rightarrow False) \Rightarrow False))) \tag{53}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p~V0p) \Leftrightarrow ($$
$$(p~V1q) \Leftrightarrow (p~V2r))) \Leftrightarrow (((p~V0p) \vee ((p~V1q) \vee (p~V2r))) \wedge (((p~V0p) \vee ((\neg($$
$$p~V2r)) \vee (\neg(p~V1q)))) \wedge (((p~V1q) \vee ((\neg(p~V2r)) \vee (\neg(p~V0p)))) \wedge ((p~V2r) \vee$$
$$((\neg(p~V1q)) \vee (\neg(p~V0p)))))))))))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p~V0p) \Leftrightarrow ($$
$$(p~V1q) \wedge (p~V2r))) \Leftrightarrow (((p~V0p) \vee ((\neg(p~V1q)) \vee (\neg(p~V2r)))) \wedge (((p~V1q) \vee$$
$$(\neg(p~V0p))) \wedge ((p~V2r) \vee (\neg(p~V0p)))))))))) \tag{55}$$

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Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge$$
$$((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))$$

$$(56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee$$
$$(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))$$

$$(57)$$

Theorem 1

$$(\forall V0f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}).(\forall V1m \in$$
$$ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V2n))\Rightarrow((ap\ (ap\ (\mathsf{c\_2Eiterate\_2Ensum}\ ty\_2Enum\_2Enum)\ (ap\ ($$
$$ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}\ V1m)\ V2n))\ V0f) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ V0f\ V1m))\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Ensum}\ ty\_2Enum\_2Enum)\ (ap\ ($$
$$ap\ c\_2Eiterate\_2E\_2E\_2E\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1m)\ (ap$$
$$\mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))$$
$$V2n))\ V0f)))))))$$