

# thm\_2Eiterate\_2ENSUM\_DEGENERATE (TMaci6CnkZYJQq6m6Aa4tAcRv7bEchVjJdq)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 7** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \tag{3}$$

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$  **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge P x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 11** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda 27a.(\lambda V2t2 \in A.\lambda 27a.$

**Definition 12** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2)) (\lambda V2t \in 2.$

**Definition 13** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A.\lambda 27a : \iota.\lambda V0x \in A.\lambda 27a.\lambda V1s \in (2^{A-27a}).(ap (c\_2Ebool\_2E\_21)$

**Definition 14** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.\lambda 27a.c\_2Ebool\_2E\_2F).$

**Definition 15** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A.\lambda 27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E\_21) 2).$

**Definition 16** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda V0f \in ((A.\lambda 27a^{A-27a})^{A-27b}).\lambda V$

**Definition 17** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A.\lambda 27a : \iota.\lambda V0op \in ((A.\lambda 27a^{A-27a})^{A-27a}).(ap (c\_2Emin\_2E\_40)$

**Definition 18** We define  $c\_2Eiterate\_2ESupport$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda V0op \in ((A.\lambda 27b^{A-27b})^{A-27b}).\lambda V$

**Definition 19** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda V0op \in ((A.\lambda 27b^{A-27b})^{A-27b}).\lambda V$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

**Definition 20** We define  $c\_2Eiterate\_2EEnum$  to be  $\lambda A.\lambda 27a : \iota.(ap (c\_2Eiterate\_2Eiterate A.\lambda 27a ty\_2Enum\_2Enum)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \tag{6}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{7}$$

**Definition 21** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Assume the following.

$$True \tag{8}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ & V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & ((ap (c\_2Eiterate\_2Eneutral \ ty\_2Enum\_2Enum) c\_2Earithmetic\_2E\_2B) = \\ & c\_2Enum\_2E0) \end{aligned} \quad (18)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0f \in (ty\_2Enum\_2Enum^{A_{27a}}). \\ & (\forall V1s \in (2^{A_{27a}}). ((\neg(p (ap (c\_2Epred\_set\_2EFINITE A_{27a}) \\ & (ap (c\_2Epred\_set\_2EGSPEC A_{27a} A_{27a}) (\lambda V2x \in A_{27a}. (ap ( \\ & ap (c\_2Epair\_2E\_2C A_{27a} 2) V2x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap \\ & (ap (c\_2Ebool\_2EIN A_{27a} V2x) V1s)) (ap c\_2Ebool\_2E\_7E (ap (ap \\ & (c\_2Emin\_2E\_3D ty\_2Enum\_2Enum) (ap V0f V2x)) c\_2Enum\_2E0)))))))))) \Rightarrow \\ & ((ap (ap (c\_2Eiterate\_2Ensum A_{27a} V1s) V0f) = c\_2Enum\_2E0))) \end{aligned}$$