thm_2Eiterate_2ENSUM__EQ__GENERAL (TMZRXF1fDdrjpVVC6zpCmvYHqjiixdmVNDh)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 10 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}$

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 12 *We define* c_2Eiterate_2Eneutral *to be* $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Emin}$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 13 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (3)$$

Definition 14 *We define* c_2Eiterate_2Esupport *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a:\iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 17 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a:\iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 18 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a:\iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 19 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a:\iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (2$

Definition 20 *We define* c_2Eiterate_2EITSET *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

Definition 21 *We define* c_2Eiterate_2Eiterate *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (4)$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Definition 22 *We define* c_2Eiterate_2Ensum *to be* $\lambda A\_27a:\iota.(ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ ty\_2Enum\_$

Definition 23 *We define* c_2Eiterate_2Emonoidal *to be* $\lambda A\_27a:\iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (ap\ \mathsf{c\_2}$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0op \in ((A\_27c^{A\_27c})^{A\_27c}).((p\ (ap\ (\mathsf{c\_2Eiterate\_2Emonoidal}\ A\_27c)\ V0op)) \Rightarrow (\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27b}). (\forall V3f \in (A\_27c^{A\_27a}).(\forall V4g \in (A\_27c^{A\_27b}).(\forall V5h \in (A\_27b^{A\_27a}).(((\forall V6y \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V6y)\ V2t)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ A\_27a)\ (\lambda V7x \in A\_27a. (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V7x)\ V1s)) (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27b)\ (ap\ V5h\ V7x))\ V6y))))))) \land (\forall V8x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V8x)\ V1s)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V5h\ V8x))\ V2t)) \land ((ap\ V4g\ (ap\ V5h\ V8x)) = (ap\ V3f\ V8x)))))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27c) V0op)\ V1s)\ V3f) = (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27b\ A\_27c) V0op)\ V2t)\ V4g)))))))))) \tag{8}$$

Assume the following.

$$(p\ (ap\ (\mathsf{c\_2Eiterate\_2Emonoidal}\ ty\_2Enum\_2Enum)\ c\_2Earithmetic\_2E\_2B)) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27b}).(\forall V2f \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V3g \in (ty\_2Enum\_2Enum^{A\_27b}). (\forall V4h \in (A\_27b^{A\_27a}).(((\forall V5y \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V5y)\ V1t)) \Rightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ A\_27a) (\lambda V6x \in A\_27a.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6x)\ V0s))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27b)\ (ap\ V4h\ V6x))\ V5y))))))) \land (\forall V7x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V7x)\ V0s)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V4h\ V7x))\ V1t)) \land ((ap\ V3g\ (ap\ V4h\ V7x)) = (ap\ V2f\ V7x)))))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Eiterate\_2Ensum}\ A\_27a) V0s)\ V2f) = (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Ensum}\ A\_27b)\ V1t)\ V3g))))))))$$