

thm_2Eiterate_2ENSUM__EQ__GENERAL__INVERSES (TMGHvx52zW6xZp7AqWJ9aYoHL2S1Spx7pRC)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2E$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E$

Definition 7 We define $c_2Ebool_2E_2EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Emin$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 12 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 15 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2E$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 17 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2)$

Definition 18 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$

Definition 19 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (4)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Definition 20 We define $c_2Eiterate_2Eenum$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate A_27a ty_2Enum_2Enum$

Definition 21 We define $c_2Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (ap c_2E$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \quad (7) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0op \in ((A.27c^{A.27c})^{A.27c}).((p\ (ap\ (c.2Eiterate.2Emonoidal \\
& \quad A.27c)\ V0op)) \Rightarrow (\forall V1s \in (2^{A.27a}).(\forall V2t \in (2^{A.27b}). \\
& \quad (\forall V3f \in (A.27c^{A.27a}).(\forall V4g \in (A.27c^{A.27b}).(\forall V5h \in \\
& \quad (A.27b^{A.27a}).(\forall V6k \in (A.27a^{A.27b}).((\forall V7y \in A.27b. \\
& \quad ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V7y)\ V2t)) \Rightarrow ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& \quad A.27a)\ (ap\ V6k\ V7y))\ V1s)) \wedge ((ap\ V5h\ (ap\ V6k\ V7y)) = V7y)))) \wedge (\forall V8x \in \\
& \quad A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V8x)\ V1s)) \Rightarrow ((p\ (ap\ (ap\ (\\
& \quad c.2Ebool.2EIN\ A.27b)\ (ap\ V5h\ V8x))\ V2t)) \wedge (((ap\ V6k\ (ap\ V5h\ V8x)) = \\
& \quad V8x) \wedge ((ap\ V4g\ (ap\ V5h\ V8x)) = (ap\ V3f\ V8x)))))) \Rightarrow ((ap\ (ap\ (ap\ (c.2Eiterate.2Eiterate \\
& \quad A.27a\ A.27c)\ V0op)\ V1s)\ V3f) = (ap\ (ap\ (ap\ (c.2Eiterate.2Eiterate \\
& \quad A.27b\ A.27c)\ V0op)\ V2t)\ V4g)))))))))
\end{aligned} \tag{8}$$

Assume the following.

$$(p\ (ap\ (c.2Eiterate.2Emonoidal\ ty.2Enum.2Enum)\ c.2Earithmetic.2E.2B)) \tag{9}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A.27a}).(\forall V1t \in (2^{A.27b}).(\forall V2f \in \\
& \quad (ty.2Enum.2Enum^{A.27a}).(\forall V3g \in (ty.2Enum.2Enum^{A.27b}). \\
& \quad (\forall V4h \in (A.27b^{A.27a}).(\forall V5k \in (A.27a^{A.27b}).((\forall V6y \in \\
& \quad A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V6y)\ V1t)) \Rightarrow ((p\ (ap\ (ap\ (\\
& \quad c.2Ebool.2EIN\ A.27a)\ (ap\ V5k\ V6y))\ V0s)) \wedge ((ap\ V4h\ (ap\ V5k\ V6y)) = \\
& \quad V6y)))) \wedge (\forall V7x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\
& \quad V7x)\ V0s)) \Rightarrow ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ (ap\ V4h\ V7x))\ V1t)) \wedge \\
& \quad (((ap\ V5k\ (ap\ V4h\ V7x)) = V7x) \wedge ((ap\ V3g\ (ap\ V4h\ V7x)) = (ap\ V2f\ V7x)))))) \Rightarrow \\
& \quad ((ap\ (ap\ (c.2Eiterate.2Eenum\ A.27a)\ V0s)\ V2f) = (ap\ (ap\ (c.2Eiterate.2Eenum \\
& \quad A.27b)\ V1t)\ V3g)))))))))
\end{aligned}$$