thm_2Eiterate_2ENSUM__IMAGE (TML8MJtoSi8hfPgqSyaXLtckxSLt9SUmmVt)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

Definition 7 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (1)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (2)$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (3)$$

Definition 11 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in(A\_27b^{A\_27a}).\lambda V1s\in$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P\in 2^{A}.$**if** $(\exists x\in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x\in A\wedge$ *of type* $\iota\Rightarrow\iota$.

Definition 13 *We define* c_2Eiterate_2Eneutral *to be* $\lambda A\_27a:\iota.\lambda V0op\in((A\_27a^{A\_27a})^{A\_27a}).(ap$ (c_2Emin

Definition 14 *We define* c_2Eiterate_2Esupport *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0op\in((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a:\iota.(\lambda V0t\in 2.(\lambda V1t1\in A\_27a.(\lambda V2t2\in A\_27a.($

Definition 16 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t\in$

Definition 17 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a:\iota.\lambda V0x\in A\_27a.\lambda V1s\in(2^{A\_27a}).(ap\ (c\_$

Definition 18 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a:\iota.(\lambda V0x\in A_27a.$c_2Ebool_2EF).

Definition 19 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 20 *We define* c_2Eiterate_2EITSET *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

Definition 21 *We define* c_2Eiterate_2Eiterate *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0op\in((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

Let $ty\_2Enum\_2Enum:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

Let $c\_2Earithmetic\_2E\_2B:\iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B\in((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Definition 22 *We define* c_2Eiterate_2Ensum *to be* $\lambda A\_27a:\iota.(ap$ (c_2Eiterate_2Eiterate $A\_27a\ ty\_2Enum\_$

Definition 23 *We define* c_2Eiterate_2Emonoidal *to be* $\lambda A\_27a:\iota.\lambda V0op\in((A\_27a^{A\_27a})^{A\_27a}).(ap\ (ap$ c_2

Assume the following.

$$True \tag{6}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t)))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0op \in ((A\_27c^{A\_27c})^{A\_27c}).((p\ (ap\ (\mathsf{c\_2Eiterate\_2Emonoidal}\ A\_27c)\ V0op)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}).(\forall V2g \in (A\_27c^{A\_27b}).\ (\forall V3s \in (2^{A\_27a}).((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.\ (((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V4x)\ V3s)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5y)\ V3s)) \wedge ((ap\ V1f\ V4x) = (ap\ V1f\ V5y)))) \Rightarrow (V4x = V5y)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27b\ A\_27c)\ V0op)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V1f)\ V3s))\ V2g) = (ap\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ A\_27c)\ V0op)\ V3s)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27c\ A\_27b)\ V2g)\ V1f)))))))))) \tag{8}$$

Assume the following.

$$(p\ (ap\ (\mathsf{c\_2Eiterate\_2Emonoidal}\ ty\_2Enum\_2Enum)\ c\_2Earithmetic\_2E\_2B)) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\ \forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (ty\_2Enum\_2Enum^{A\_27b}).\ (\forall V2s \in (2^{A\_27a}).((\forall V3x \in A\_27a.(\forall V4y \in A\_27a.\ (((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V2s)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V4y)\ V2s)) \wedge ((ap\ V0f\ V3x) = (ap\ V0f\ V4y)))) \Rightarrow (V3x = V4y)))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Eiterate\_2Ensum}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V0f)\ V2s))\ V1g) = (ap\ (ap\ (\mathsf{c\_2Eiterate\_2Ensum}\ A\_27a)\ V2s)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ ty\_2Enum\_2Enum\ A\_27b)\ V1g)\ V0f)))))))$$