

thm_2Eiterate_2ENSUM__INCL__EXCL (TM- chvvM4XoTaqXNZHWm9Rjpumw8aZJDZsqj)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 10 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2)$

Definition 11 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2)))$

Definition 12 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2)$

Definition 13 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2)$

Definition 14 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 15 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2)$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Emin_2E_40) A_27a)$

Definition 18 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V1x \in A_27b$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_21) 2) t1 t2)))$

Definition 20 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V1x \in A_27b$

Definition 21 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V1x \in A_27b$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Definition 22 We define $c_2Eiterate_2Eenum$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate A_27a ty_2Enum_2Enum) A_27a)$

Definition 23 We define $c_2Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (ap c_2Eiterate_2Eenum) A_27a)$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (7)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\
& \forall V0op \in ((A_{27a}^{A_{27a}})^{A_{27a}}).((p\ (ap\ (c_2Eiterate_2Emonoidal \\
& \quad A_{27a}\ V0op))) \Rightarrow (\forall V1s \in (2^{A_{27b}}).(\forall V2t \in (2^{A_{27b}}). \\
& \quad (\forall V3f \in (A_{27a}^{A_{27b}}).(((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad A_{27b}\ V1s)) \wedge (p\ (ap\ (c_2Epred_set_2EFINITE\ A_{27b}\ V2t)))) \Rightarrow ((\\
& \quad ap\ (ap\ V0op\ (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_{27b}\ A_{27a}\ V0op) \\
& \quad V1s)\ V3f))\ (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_{27b}\ A_{27a}\ V0op) \\
& \quad V2t)\ V3f)) = (ap\ (ap\ V0op\ (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_{27b} \\
& \quad A_{27a}\ V0op)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_{27b}\ V1s)\ V2t))\ V3f)) \\
& \quad (ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_{27b}\ A_{27a}\ V0op)\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad A_{27b}\ V1s)\ V2t))\ V3f))))))))) \\
& \hspace{15em} (8)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (c_2Eiterate_2Emonoidal\ ty_2Enum_2Enum)\ c_2Earithmetic_2E_2B)) \\
& \hspace{15em} (9)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in \\
& (2^{A_{27a}}).(\forall V2f \in (ty_2Enum_2Enum^{A_{27a}}).(((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad A_{27a}\ V0s)) \wedge (p\ (ap\ (c_2Epred_set_2EFINITE\ A_{27a}\ V1t)))) \Rightarrow ((\\
& \quad ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ (c_2Eiterate_2Ensum\ A_{27a} \\
& \quad V0s)\ V2f))\ (ap\ (ap\ (c_2Eiterate_2Ensum\ A_{27a}\ V1t)\ V2f)) = (ap\ (ap \\
& \quad c_2Earithmetic_2E_2B\ (ap\ (ap\ (c_2Eiterate_2Ensum\ A_{27a}\ (ap\ (\\
& \quad ap\ (c_2Epred_set_2EUNION\ A_{27a}\ V0s)\ V1t))\ V2f))\ (ap\ (ap\ (c_2Eiterate_2Ensum \\
& \quad A_{27a}\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_{27a}\ V0s)\ V1t))\ V2f)))))))))
\end{aligned}$$