

thm_2Eiterate_2ENSUM__SUPERSET (TMGn-JptGEpQ26KS2mwYpUEXHKFdmi5Txb9Q)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set (V0s, V1t)) (\lambda V2t \in 2. inj_o (V1t = V2t)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V1t2 = V2t))))$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x)) \text{ else } (\lambda x. x \in A \wedge \neg p x)$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota. \lambda V0op \in ((A_27a^{A_27a})^{A_27a}). (ap (c_2Emin_2E_40 (V0op)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty_2Epair_2Eprod \\ & \quad A0 \ A1) \end{aligned} \tag{1}$$

Let $c_2Epair_2EAABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow c_2Epair_2EAABS_prod \\ & \quad A_27a \ A_27b \in ((ty_2Epair_2Eprod \ A_27a \ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \tag{2}$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2Eprod A_27a A_27b) ((ty_2Epair_2Eprod A_27a A_27b) (A_27a A_27b)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})((ty_2Epred_set A_27a A_27b) (A_27a A_27b))) \end{aligned} \quad (3)$$

Definition 13 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V1$

Definition 14 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap (c_2Ebool_2Eprod A_27a A_27b) ((V0t V1t1) (V2t2))))))$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2Eprod A_27a A_27b) ((V0t1 V1t2) (V2t))))))$

Definition 16 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Epred_set A_27a A_27b) ((V0x V1s) (A_27a A_27b)))$

Definition 17 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF))$

Definition 18 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2Eprod A_27a A_27b) ((V0s V2t) (A_27a A_27b)))$

Definition 19 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27a})^{A_27b}). \lambda V1$

Definition 20 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V1$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (4)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Definition 21 We define $c_2Eiterate_2Ensum$ to be $\lambda A_27a : \iota. (ap (c_2Eiterate_2Eiterate A_27a ty_2Enum_2Enum) ((V0x V1s) (A_27a A_27b)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^omega) \quad (7)$$

Definition 22 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 23 We define $c_2Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota. \lambda V0op \in ((A_27a^{A_27a})^{A_27a}). (ap (ap (c_2Eiterate_2Eiterate A_27a A_27b) ((V0op V1s) (A_27a A_27b))) ((V0op V1s) (A_27a A_27b)))$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (10)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (13)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (14)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\ & \forall V0op \in ((A_27b^{A_27b})^{A_27b}).((p (ap (c_2Eiterate_2Emonoidal \\ & A_27b) V0op)) \Rightarrow (\forall V1f \in (A_27b^{A_27a}).(\forall V2u \in (2^{A_27a}). \\ & (\forall V3v \in (2^{A_27a}).(((p (ap (ap (c_2Epred_set_2ESUBSET \\ & A_27a) V2u) V3v)) \wedge (\forall V4x \in A_27a.((p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V4x) V3v)) \wedge (\neg(p (ap (ap (c_2Ebool_2EIN A_27a) V4x) V2u)))))) \Rightarrow \\ & ((ap V1f V4x) = (ap (c_2Eiterate_2Eneutral A_27b) V0op)))))) \Rightarrow ((\\ & ap (ap (ap (c_2Eiterate_2Eiterate A_27a A_27b) V0op) V3v) V1f) = \\ & (ap (ap (ap (c_2Eiterate_2Eiterate A_27a A_27b) V0op) V2u) V1f))))))) \end{aligned} \quad (20)$$

Assume the following.

$$((ap (c_2Eiterate_2Eneutral ty_2Enum_2Enum) c_2Earithmetic_2E_2B) = \\ c_2Enum_2E0) \quad (21)$$

Assume the following.

$$(p (ap (c_2Eiterate_2Emonoidal ty_2Enum_2Enum) c_2Earithmetic_2E_2B)) \quad (22)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0f \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1u \in (2^{A_27a}).(\forall V2v \in (2^{A_27a}).(((p (ap (ap (\\ & c_2Epred_set_2ESUBSET A_27a) V1u) V2v)) \wedge (\forall V3x \in A_27a. \\ & (((p (ap (ap (c_2Ebool_2EIN A_27a) V3x) V2v)) \wedge (\neg(p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V3x) V1u)))))) \Rightarrow ((ap V0f V3x) = c_2Enum_2E0)))) \Rightarrow ((ap (ap (c_2Eiterate_2Ensum \\ & A_27a) V2v) V0f) = (ap (ap (c_2Eiterate_2Ensum A_27a) V1u) V0f))))))) \end{aligned}$$