

thm_2Eiterate_2ENSUM_SUPPORT

(TMRW5V6FufKKbMtHFuCaFLBweayqMm4Ndcj)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (P \Rightarrow Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) (\text{c_2Ebool_2EF } 2)))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P \text{ (ap } P \text{ x)}) \text{ then } (\lambda x. x \in A \wedge P \text{ x})$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Eiterate_2Eneutral` to be $\lambda A. \lambda 27a : \iota. \lambda V0op \in ((A-27a)^{A-27a})^{A-27a}. (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0op))$

Definition 10 We define `c_2Ebool_2EIN` to be $\lambda A. \lambda 27a : \iota. (\lambda V0x \in A-27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \text{ } V0x)))$

Definition 11 We define `c_2Ebool_2ECOND` to be $\lambda A. \lambda 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A-27a. (\lambda V2t2 \in A-27a. (\text{ap } V2t2 \text{ } V1t1))))$

Definition 12 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 \text{ } A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A-27a. \text{nonempty } A-27a \Rightarrow \forall A-27b. \text{nonempty } A-27b \Rightarrow \text{c_2Epair_2EABS_prod } A-27a \text{ } A-27b \in ((\text{ty_2Epair_2Eprod } A-27a \text{ } A-27b))^{((2^{A-27b})^{A-27a})} \tag{2}$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(3)

Definition 14 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Ebool_2E21$

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 16 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E21$

Definition 17 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27a})^{A_27b}). \lambda V$

Definition 18 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(4)

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$
(5)

Definition 19 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V$

Definition 20 We define $c_2Eiterate_2Esum$ to be $\lambda A_27a : \iota. (ap (c_2Eiterate_2Eiterate A_27a ty_2Enum_2Enum$

Assume the following.

$$True$$
(6)

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t)))$$
(7)

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True))$$
(8)

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x))))$$
(9)

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0op \in ((A_27a^{A_27a})^{A_27a}). (\forall V1f \in (A_27a^{A_27b}). \\ & (\forall V2s \in (2^{A_27b}). ((ap (ap (ap (c_2Eiterate_2Esupport A_27b \\ & A_27a) V0op) V1f) (ap (ap (ap (c_2Eiterate_2Esupport A_27b A_27a) \\ & V0op) V1f) V2s)) = (ap (ap (ap (c_2Eiterate_2Esupport A_27b A_27a) \\ & V0op) V1f) V2s)))))) \end{aligned} \quad (12)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1s \in (2^{A_27a}). ((ap (ap (c_2Eiterate_2Enum A_27a) (\\ & ap (ap (ap (c_2Eiterate_2Esupport A_27a ty_2Enum_2Enum) c_2Earithmetic_2E_2B) \\ & V0f) V1s)) V0f) = (ap (ap (c_2Eiterate_2Enum A_27a) V1s) V0f)))) \end{aligned}$$