

thm_2Eiterate_2ENSUM__SWAP (TMQnKhzb- sPhwFfrhBwG8qHxzB4SCkD81Ba5)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_2K$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b.V0x))$

Definition 4 We define $c_2Ecombin_2E_2S$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2E_2I$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2E_2S A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)) P))$

Definition 7 We define $c_2Ecombin_2E_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27a}). (ap (ap (c_2Ecombin_2E_2K A_27a A_27b) V0f) V1g)$

Definition 8 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (P \Rightarrow Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2)) (V0t1 \wedge V1t2)))$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_2COND$ to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V0t \wedge (ap (c_2Emin_2E_40 A_27a) V1t1) V2t2))))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 14 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota. \lambda V0op \in ((A_27a^{A_27a})^{A_27a}). (ap\ (c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E)$

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 16 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{5}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{6}$$

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \tag{7}$$

Definition 18 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V1f \in (2^{A_27b}). (ap\ (c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E)))))$

Definition 20 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 21 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_7E)$.

Definition 22 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V1t \in 2. (ap\ (c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E)))$

Definition 23 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27a})^{A_27b}). \lambda V1g \in (2^{A_27b}). (ap\ (c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 24 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V1f \in (2^{A_27b}). (ap\ (c_2Emin\ 2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 25 We define $c_2Eiterate_2Eenum$ to be $\lambda A_27a : \iota. (ap\ (c_2Eiterate_2Eiterate\ A_27a\ ty_2Enum_2Enum^{\omega}))$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2B V0m) V2p) = (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) \Leftrightarrow (V0m = V1n)))))) \quad (8)$$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (11)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (14)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg (\neg (p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (15)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (V0x = V0x)) \quad (16)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\forall V3x \in A_27a. (p (ap V1Q V3x))))))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (22)$$

Assume the following.

$$2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) V5y_27)))))))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (((ap (ap (c_2Ecombin_2Eo A_27a A_27b) A_27b) (c_2Ecombin_2EI A_27b)) V0f) = V0f) \wedge ((ap (ap (c_2Ecombin_2Eo A_27a A_27b A_27a) V0f) (c_2Ecombin_2EI A_27a)) = V0f))) \quad (26)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (ty_2Enum_2Enum^{A_27a}).((ap\ (ap\ (c_2Eiterate_2Enum \\
& A_27a)\ (c_2Epred_set_2EEMPTY\ A_27a))\ V0f) = c_2Enum_2E0)) \wedge (\\
& \forall V1x \in A_27b.(\forall V2f \in (ty_2Enum_2Enum^{A_27b}).(\forall V3s \in \\
& (2^{A_27b}).((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V3s)) \Rightarrow ((ap \\
& (ap\ (c_2Eiterate_2Enum\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& A_27b)\ V1x)\ V3s))\ V2f) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Enum_2Enum) \\
& (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V1x)\ V3s))\ (ap\ (ap\ (c_2Eiterate_2Enum \\
& A_27b)\ V3s)\ V2f))\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ V2f\ V1x))\ (ap \\
& (ap\ (c_2Eiterate_2Enum\ A_27b)\ V3s)\ V2f))))))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Enum_2Enum^{A_27a}). \\
& (\forall V1g \in (ty_2Enum_2Enum^{A_27a}).(\forall V2s \in (2^{A_27a}). \\
& ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V2s)) \Rightarrow ((ap\ (ap\ (c_2Eiterate_2Enum \\
& A_27a)\ V2s)\ (\lambda V3x \in A_27a.(ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ V0f \\
& V3x))\ (ap\ V1g\ V3x)))) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ (c_2Eiterate_2Enum \\
& A_27a)\ V2s)\ V0f))\ (ap\ (ap\ (c_2Eiterate_2Enum\ A_27a)\ V2s)\ V1g)))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((ap\ (ap \\
& (c_2Eiterate_2Enum\ A_27a)\ V0s)\ (\lambda V1n \in A_27a.c_2Enum_2E0)) = \\
& \hspace{15em} c_2Enum_2E0)) \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}).((\\
& (p\ (ap\ V0P\ (c_2Epred_set_2EEMPTY\ A_27a))) \wedge (\forall V1s \in (2^{A_27a}). \\
& (((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\
& (\forall V2e \in A_27a.((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2e)\ V1s))) \Rightarrow \\
& (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2e)\ V1s)))))) \Rightarrow \\
& (\forall V3s \in (2^{A_27a}).((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ \\
& V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{15em} (31)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \hspace{15em} (32)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \hspace{15em} (33)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (35)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ & \forall V0f \in ((ty.2Enum.2Enum^{A.27b})^{A.27a}). (\forall V1s \in (2^{A.27a}). \\ & (\forall V2t \in (2^{A.27b}). (((p (ap (c.2Epred_set.2EFINITE A.27a) \\ & V1s)) \wedge (p (ap (c.2Epred_set.2EFINITE A.27b) V2t))) \Rightarrow ((ap (ap (\\ & c.2Eiterate.2Ensum A.27a) V1s) (\lambda V3i \in A.27a. (ap (ap (c.2Eiterate.2Ensum \\ & A.27b) V2t) (ap V0f V3i)))) = (ap (ap (c.2Eiterate.2Ensum A.27b) \\ & V2t) (\lambda V4j \in A.27b. (ap (ap (c.2Eiterate.2Ensum A.27a) V1s) (\\ & \lambda V5i \in A.27a. (ap (ap V0f V5i) V4j)))))))))) \end{aligned}$$