

# thm\_2Eiterate\_2EPOLYNOMIAL\_\_FUNCTION\_\_o (TMR73dmyquDkRaEmW6wY4xkVWixDR7ayhQQ)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))))$

**Definition 4** We define  $c\_2Ecombin\_2E\_o$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.\lambda V0f \in (A.\lambda b^{A-27c}).\lambda V1g$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{1}$$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{2}$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealx\_2Ereal^{ty\_2Eenum\_2Eenum})^{ty\_2Erealx\_2Ereal}) \tag{3}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{4}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{5}$$

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal}) \tag{6}$$

**Definition 5** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 6** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) \quad (7)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} \quad (9)$$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$

**Definition 8** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 9** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 10** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (13)$$

**Definition 14** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 15** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 16** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 17** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 18** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (14)$$

**Definition 19** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A-27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A-27b})}) \end{aligned} \quad (15)$$

**Definition 20** We define  $c\_2Eiterate\_2E\_2E\_2E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Erealax\_2Etreax\_add : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etreax\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal) \end{aligned} \quad (16)$$

**Definition 21** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 22** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota. \lambda V0op \in ((A\_27a^{A-27a})^{A-27a}). (ap\ (c\_2Emin$

**Definition 23** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A-27a}). (ap\ V1f\ V0x))$

**Definition 24** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0op \in ((A\_27b^{A-27b})^{A-27b}). \lambda V$

**Definition 25** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 26** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A-27a}). (ap\ (c\_2$

**Definition 27** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF).$

**Definition 28** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). (ap\ (c\_2Ebool\_2E\_21\ 2)$

**Definition 29** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in ((A\_27a^{A-27a})^{A-27b}). \lambda V$

**Definition 30** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0op \in ((A\_27b^{A-27b})^{A-27b}). \lambda V$

**Definition 31** We define  $c\_2\text{Eiterate\_2ESum}$  to be  $\lambda A\_27a : \iota.(ap (c\_2\text{Eiterate\_2Eiterate } A\_27a \text{ ty\_2Erealax}$

**Definition 32** We define  $c\_2\text{Eiterate\_2Epolynomial\_function}$  to be  $\lambda V0p \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax}.$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \wedge (p \ V1t2)) \Leftrightarrow ((p \ V1t2) \wedge (p \ V0t1)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg (p \ V0t)))) \quad (22)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))) \quad (26)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{27a}}). ((\forall V2x \in A_{27a}. ((p \ V0P) \Rightarrow (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \ V0P) \Rightarrow (\forall V3x \in A_{27a}. (p \ (ap \ V1Q \ V3x))))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (28)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p \ V0x) \Leftrightarrow (p \ V1x_{27})) \wedge ((p \ V1x_{27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{27})))))) \Rightarrow ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{27}) \Rightarrow (p \ V3y_{27})))) \quad (29)$$

Assume the following.

$$(\forall V0c \in \text{ty\_2Erealax\_2Ereal}. (p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \ (\lambda V1x \in \text{ty\_2Erealax\_2Ereal}. V0c)))) \quad (30)$$

Assume the following.

$$(\forall V0p \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}}). (\forall V1q \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}}). (((p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \ V0p)) \wedge (p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \ V1q))) \Rightarrow (p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \ (\lambda V2x \in \text{ty\_2Erealax\_2Ereal}. (ap \ (ap \ c\_2Erealax\_2Ereal\_add \ (ap \ V0p \ V2x)) \ (ap \ V1q \ V2x))))))) \quad (31)$$

Assume the following.

$$(\forall V0p \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}}). (\forall V1q \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}}). (((p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \ V0p)) \wedge (p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \ V1q))) \Rightarrow (p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \ (\lambda V2x \in \text{ty\_2Erealax\_2Ereal}. (ap \ (ap \ c\_2Erealax\_2Ereal\_mul \ (ap \ V0p \ V2x)) \ (ap \ V1q \ V2x))))))) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})})), \\
& (((p (ap V0P (\lambda V1x \in ty\_2Erealax\_2Ereal.V1x))) \wedge ((\forall V2c \in \\
& ty\_2Erealax\_2Ereal.(p (ap V0P (\lambda V3x \in ty\_2Erealax\_2Ereal. \\
& V2c)))) \wedge ((\forall V4p \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& (\forall V5q \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(((p \\
& (ap V0P V4p)) \wedge (p (ap V0P V5q))) \Rightarrow (p (ap V0P (\lambda V6x \in ty\_2Erealax\_2Ereal. \\
& (ap (ap c\_2Erealax\_2Ereal\_add (ap V4p V6x)) (ap V5q V6x)))))) \wedge \\
& (\forall V7p \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V8q \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(((p (ap V0P V7p)) \wedge \\
& (p (ap V0P V8q))) \Rightarrow (p (ap V0P (\lambda V9x \in ty\_2Erealax\_2Ereal.(ap ( \\
& ap c\_2Erealax\_2Ereal\_mul (ap V7p V9x)) (ap V8q V9x)))))) \Rightarrow \\
& (\forall V10p \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).((p \\
& (ap c\_2Eiterate\_2Epolynomial\_function V10p)) \Rightarrow (p (ap V0P V10p))))))
\end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{43}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{44}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{45}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0p \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1q \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (((p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \\
& V0p)) \wedge (p \ (ap \ c\_2Eiterate\_2Epolynomial\_function \ V1q))) \Rightarrow (p \ ( \\
& ap \ c\_2Eiterate\_2Epolynomial\_function \ (ap \ (ap \ (c\_2Ecombin\_2Eo \\
& ty\_2Erealax\_2Ereal \ ty\_2Erealax\_2Ereal \ ty\_2Erealax\_2Ereal) \\
& V0p) \ V1q))))))
\end{aligned}$$