

thm_2Eiterate_2EREAL__SUP__EQ__INF
(TMGhF8W59aa3FzrxotzKzjWUqbfFvCnAWcH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p (ap P x))$) of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})$

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Emin_2E_40$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \tag{5}$$

Definition 7 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Definition 11 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_21) 2) (\lambda V2t2 \in 2.V2t2))))$

Definition 14 We define c_2Ereal_2Emax to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 15 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E_40) ty_2Erealax_2Ereal)$

Definition 16 We define c_2Ereal_2Emin to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 17 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 18 We define $c_2Eiterate_2Einf$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E_40) ty_2Erealax_2Ereal)$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 20 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod) A_27a A_27b)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \end{aligned} \quad (7)$$

Definition 21 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Epair_2E_2C) A_27a A_27a V0x V1s)$

Definition 22 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 23 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2) V0s$

Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (8)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (10)$$

Definition 24 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 25 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (13)$$

Definition 26 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (14)$$

Definition 27 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONJ$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge (p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow \text{False}) \Rightarrow (\neg(p \ V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow \text{False}))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge \text{True}) \Leftrightarrow (p \ V0t)) \wedge (((\text{False} \wedge (p \ V0t)) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \vee (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \vee \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \text{False}) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg(p \ V0t)))))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge ((\neg \text{False}) \Leftrightarrow \text{True}))) \quad (25)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \text{True})) \quad (26)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p \ V0t)))))) \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in \\ A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in \\ A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ 2. ((\exists V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \vee (p\ V1Q)) \Leftrightarrow (\exists V3x \in \\ A_27a. ((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((p\ V0P) \vee (\exists V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((\exists V2x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (41)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))) \quad (42)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0P \in ((2^{A_27b})^{A_27a}). ((\forall V1x \in A_27a. (\exists V2y \in A_27b. (p\ (ap\ (ap\ V0P\ V1x)\ V2y))) \Leftrightarrow (\exists V3f \in (A_27b^{A_27a}). (\forall V4x \in A_27a. (p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \quad (45)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(((\neg(V0s = (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal)))) \wedge (\exists V1b \in ty_2Erealax_2Ereal.(\forall V2x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V2x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V2x) V1b)))))) \Rightarrow ((\forall V3x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V3x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V3x) (ap c_2Ereal_2Esup \\
& V0s)))))) \wedge (\forall V4b \in ty_2Erealax_2Ereal.((\forall V5x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x) V0s)) \Rightarrow (p (\\
& ap (ap c_2Ereal_2Ereal_lte V5x) V4b)))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad (ap c_2Ereal_2Esup V0s) V4b)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(((\neg(V0s = (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal)))) \wedge (\exists V1b \in ty_2Erealax_2Ereal.(\forall V2x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V2x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V1b) V2x)))))) \Rightarrow ((\forall V3x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V3x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Eiterate_2Einf \\
& V0s) V3x)))))) \wedge (\forall V4b \in ty_2Erealax_2Ereal.((\forall V5x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V5x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V4b) V5x)))))) \Rightarrow (p (ap \\
& \quad (ap c_2Ereal_2Ereal_lte V4b) (ap c_2Eiterate_2Einf V0s)))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (c_2Epred_set_2EFINITE ty_2Erealax_2Ereal) V1s)) \Rightarrow (\\
& (ap c_2Ereal_2Esup (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) \\
& V0x) V1s)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Erealax_2Ereal) (\\
& ap (ap (c_2Emin_2E_3D (2^{ty_2Erealax_2Ereal}) V1s) (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal))) V0x) (ap (ap c_2Ereal_2Emin V0x) (ap c_2Ereal_2Esup \\
& \quad V1s)))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (c_2Epred_set_2EFINITE ty_2Erealax_2Ereal) V1s)) \Rightarrow (\\
& (ap c_2Eiterate_2Einf (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) \\
& V0x) V1s)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Erealax_2Ereal) (\\
& ap (ap (c_2Emin_2E_3D (2^{ty_2Erealax_2Ereal}) V1s) (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal))) V0x) (ap (ap c_2Ereal_2Emin V0x) (ap c_2Eiterate_2Einf \\
& \quad V1s)))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s))))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE \\ & A_27a)\ (c_2Epred_set_2EEMPTY\ A_27a))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\ & (((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & V1y)\ V0x))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1k \in ty_2Erealax_2Ereal. \\ & ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Eabs\ V0x)\ V1k)) \Leftrightarrow \\ & ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Erealax_2Ereal_neg\ V1k) \\ & V0x)) \wedge (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0x)\ V1k)))))) \end{aligned} \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (58)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (70)$$

Theorem 1

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(((\neg(V0s = (c_2Epred_set_2EEMPTY \\ & \quad ty_2Erealax_2Ereal)))) \wedge (\exists V1B \in ty_2Erealax_2Ereal.(\forall V2x \in \\ & \quad ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & \quad V2x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Eabs \\ & \quad V2x)) V1B))))))) \Rightarrow (((ap c_2Ereal_2Esup V0s) = (ap c_2Eiterate_2Einf \\ & \quad V0s)) \Leftrightarrow (\exists V3a \in ty_2Erealax_2Ereal.(V0s = (ap (ap (c_2Epred_set_2EINSERT \\ & \quad ty_2Erealax_2Ereal) V3a) (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))))))) \end{aligned}$$