

# thm\_2Eiterate\_2ESUM\_ABS\_LE (TM- FXLJQRMM3dFoNLAghkXg4cmFPCmap4ofq)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_7E$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))))$

**Definition 4** We define  $c\_2Ebool\_2E\_7E$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V 0t \in 2.V 0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V 0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V 0t) c\_2Ebool\_2E\_7E))$

**Definition 7** We define  $c\_2Ebool\_2E\_IN$  to be  $\lambda A. 27a : \iota. (\lambda V 0x \in A. 27a. (\lambda V 1f \in (2^{A-27a}). (ap V 1f V 0x)))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A 0. nonempty\ A 0 \Rightarrow \forall A 1. nonempty\ A 1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A 0\ A 1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) ty\_2Erealax) \tag{4}$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_Erealax\_Ereal\_REP$  to be  $\lambda V0a \in ty\_Erealax\_Ereal.(ap (c\_Emin\_E40 (ty\_Erealax\_Ereal\_add) (ty\_Erealax\_Ereal\_add)))$ .  
Let  $c\_Erealax\_Ereal\_add : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_add \in (((ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) \quad (5)$$

Let  $c\_Erealax\_Ereal\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_eq \in ((2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)} (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal))) (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal) \quad (6)$$

Let  $c\_Erealax\_Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_ABS\_CLASS \in (ty\_Erealax\_Ereal)^{(2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)} (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal))} \quad (7)$$

**Definition 10** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)$ .

**Definition 11** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$ .

**Definition 12** We define  $c\_Ebool\_E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E21) 2)) (\lambda V2t \in 2))$ .

**Definition 13** We define  $c\_Eiterate\_Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap (c\_Emin\_E40) (c\_Emin\_E40))$ .

Let  $c\_Epair\_EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_Epair\_EABS\_prod A\_27a A\_27b \in ((ty\_Epair\_Eprod A\_27a A\_27b)^{(2^{(A\_27b)^{A\_27a}})}) \quad (8)$$

**Definition 14** We define  $c\_Epair\_E2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_Ebool\_E21) 2)$ .

Let  $c\_Epred\_set\_EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_Epred\_set\_EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_Epair\_Eprod A\_27a 2)^{A\_27b}}) \quad (9)$$

**Definition 15** We define  $c\_Eiterate\_Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V1$ .

**Definition 16** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.))$ .

**Definition 17** We define  $c\_Ebool\_E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E21) 2)) (\lambda V2t \in 2))$ .

**Definition 18** We define  $c\_Epred\_set\_EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_Ebool\_E21) 2)$ .

**Definition 19** We define  $c\_Epred\_set\_EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_Ebool\_E2F)$ .

**Definition 20** We define  $c\_Epred\_set\_EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_Ebool\_E21) 2)$ .

**Definition 21** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$ .

**Definition 22** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$ .

**Definition 23** We define  $c\_2Eiterate\_2ESum$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eiterate\_2Eiterate A\_27a ty\_2Erealax$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \end{aligned} \quad (10)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (11)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (12)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (13)$$

**Definition 25** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (14)$$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)}) \quad (15)$$

**Definition 26** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 27** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 28** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECONJ$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow \neg (p \ V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg ( \\
& p \ V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow \\
& (p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\
& 2. (((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0f \in (ty.2Erealax.2Ereal^{A.27a}). \\
& (\forall V1g \in (ty.2Erealax.2Ereal^{A.27a}). (\forall V2s \in (2^{A.27a}). \\
& (((p \ (ap \ (c.2Epred\_set.2EFINITE \ A.27a) \ V2s)) \wedge (\forall V3x \in A.27a. \\
& ((p \ (ap \ (ap \ (c.2Ebool.2EIN \ A.27a) \ V3x) \ V2s)) \Rightarrow (p \ (ap \ (ap \ c.2Ereal.2Ereal\_lte \\
& (ap \ V0f \ V3x) \ (ap \ V1g \ V3x)))))) \Rightarrow (p \ (ap \ (ap \ c.2Ereal.2Ereal\_lte \\
& (ap \ (ap \ (c.2Eiterate.2ESum \ A.27a) \ V2s) \ V0f)) \ (ap \ (ap \ (c.2Eiterate.2ESum \\
& A.27a) \ V2s) \ V1g))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0f \in (ty.2Erealax.2Ereal^{A.27a}). \\
& (\forall V1s \in (2^{A.27a}). ((p \ (ap \ (c.2Epred\_set.2EFINITE \ A.27a) \\
& V1s)) \Rightarrow (p \ (ap \ (ap \ c.2Ereal.2Ereal\_lte \ (ap \ c.2Ereal.2Eabs \ (ap \ ( \\
& ap \ (c.2Eiterate.2ESum \ A.27a) \ V1s) \ V0f))) \ (ap \ (ap \ (c.2Eiterate.2ESum \\
& A.27a) \ V1s) \ (\lambda V2x \in A.27a. (ap \ c.2Ereal.2Eabs \ (ap \ V0f \ V2x))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z))) \Rightarrow (p (ap ( \\
& ap c\_2Ereal\_2Ereal\_lte V0x) V2z)))))) \tag{26}
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). \\
& (\forall V1g \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V2s \in (2^{A\_27a}). \\
& ((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) V2s)) \wedge (\forall V3x \in A\_27a. \\
& ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V3x) V2s)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_2Eabs (ap V0f V3x)) (ap V1g V3x)))))) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Ereal\_2Eabs (ap (ap (c\_2Eiterate\_2ESum A\_27a) V2s) V0f))) \\
& (ap (ap (c\_2Eiterate\_2ESum A\_27a) V2s) V1g))))))
\end{aligned}$$