

thm\_2Eiterate\_2ESUM\_CASES\_\_1  
(TMJx3ZQil1iSdB2yZhEGSMRYMRfVZFyqwgSx)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 4** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V 0t \in 2.V 0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V 0t1 \in 2. (\lambda V 1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V 2t \in 2.V 2t))))$

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Ebool_2ECOND` to be  $\lambda A. 27a : \iota. (\lambda V 0t \in 2. (\lambda V 1t1 \in A. 27a. (\lambda V 2t2 \in A. 27a. (\text{ap } (\text{c\_2Emin\_2E\_40 } (2^{A-27a}))))))$

Let `ty_2Ehreal_2Ehreal` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Ehreal\_2Ehreal} \tag{1}$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty\_2Epair\_2Eprod } A 0 A 1) \tag{2}$$

Let `ty_2Erealax_2Ereal` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Erealax\_2Ereal} \tag{3}$$

Let `c_2Erealax_2Ereal__REP__CLASS` :  $\iota$  be given. Assume the following.

$$\text{c\_2Erealax\_2Ereal\_REP\_CLASS} \in ((2^{(\text{ty\_2Epair\_2Eprod } \text{ty\_2Ehreal\_2Ehreal } \text{ty\_2Ehreal\_2Ehreal}) \text{ty\_2Erealax\_2Ereal}})) \tag{4}$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E.40 (ty\_2Erealax\_2Ereal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}} \quad (7)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 12** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap (c\_2Emin\_2E.40 (ty\_2Erealax\_2Ereal\_add : \iota$

**Definition 13** We define  $c\_2Ebool\_2E.7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E.3D\_3D\_3E V0t) c\_2Ebool\_2E.7E$

**Definition 14** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (8)$$

**Definition 15** We define  $c\_2Epair\_2E.2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Emin\_2E.40 (ty\_2Erealax\_2Ereal\_add : \iota$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (9)$$

**Definition 16** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V1$

**Definition 17** We define  $c\_2Ebool\_2E.5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E.21 2) (\lambda V2t \in 2$

**Definition 18** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Emin\_2E.40 (ty\_2Erealax\_2Ereal\_add : \iota$

**Definition 19** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E.2F)$ .

**Definition 20** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E.21 2)$

**Definition 21** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$ .

**Definition 22** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$ .

**Definition 23** We define  $c\_2Eiterate\_2ESum$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eiterate\_2Eiterate A\_27a ty\_2Erealax$

**Definition 24** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 25** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2$

**Definition 26** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap (ap$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{10}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{11}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{12}$$

**Definition 27** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 28** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{13}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{14}$$

**Definition 29** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

**Definition 30** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 31** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \tag{16}$$

**Definition 32** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 33** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (17)$$

**Definition 34** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (18)$$

**Definition 35** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (19)$$

**Definition 36** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27b.((ap\ (\lambda V2x \in A\_27b.V0t1)\ V1t2) = V0t1))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (25)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\
& p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1a \in A\_27a. ((\exists V2x \in A\_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V1s \in (2^{A\_27a}). (\forall V2a \in A\_27a. ((p (ap (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V1s)) \wedge (p (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V2a)\ V1s)))) \Rightarrow ((ap (c\_2Eiterate\_2ESum\ A\_27a)\ (ap (ap (c\_2Epred\_set\_2EDELETE\ A\_27a)\ V1s)\ V2a))\ V0f) = (ap (ap c\_2Ereal\_2Ereal\_sub (ap (ap (c\_2Eiterate\_2ESum\ A\_27a)\ V1s)\ V0f)) (ap\ V0f\ V2a)))))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V1x \in A\_27a. ((ap (ap (c\_2Eiterate\_2ESum\ A\_27a)\ (ap (ap (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a))))\ V0f) = (ap\ V0f\ V1x)))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1P \in (2^{A\_27a}). (\forall V2f \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V3g \in (ty\_2Erealax\_2Ereal^{A\_27a}). ((p (ap (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V0s)) \Rightarrow ((ap (ap (c\_2Eiterate\_2ESum\ A\_27a)\ V0s)\ (\lambda V4x \in A\_27a. (ap (ap (ap (c\_2Ebool\_2ECOND\ ty\_2Erealax\_2Ereal)\ (ap\ V1P\ V4x)) (ap\ V2f\ V4x)) (ap\ V3g\ V4x)))))) = (ap (ap c\_2Erealax\_2Ereal\_add (ap (ap (c\_2Eiterate\_2ESum\ A\_27a)\ (ap (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V5x \in A\_27a. (ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ 2)\ V5x)\ (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V5x)\ V0s)) (ap\ V1P\ V5x))))))\ V2f)) (ap (ap (c\_2Eiterate\_2ESum\ A\_27a)\ (ap (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V6x \in A\_27a. (ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ 2)\ V6x)\ (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V6x)\ V0s)) (ap c\_2Ebool\_2E\_7E (ap\ V1P\ V6x))))))\ V3g)))))) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in A\_27b. (((ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}). (\forall V1v \in \\ & A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2E2C \\ & A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg (p\ (ap\ (ap \\ & (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & A\_27a. (\forall V2s \in (2^{A\_27a}). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ & V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ & A\_27a)\ V1y)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & V1y)\ V0x)))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\ & (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & V0x)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V1y)\ V2z)) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y)\ V2z)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)\ V0x) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & c\_2Enum\_2E0)))) \end{aligned} \quad (47)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)))))) V0x) = V0x)) \quad (48)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_add V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x)) \quad (49)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_add V0x) (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) = (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Erealax\_2Ereal\_neg V0x)) (ap c\_2Erealax\_2Ereal\_neg V1y)))))) \quad (51)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V0x))) \quad (53)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (54)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x) V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)))))) \quad (55)$$



Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lte V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte V0y) V1x)))))) \quad (56)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)))))) \quad (57)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V0x)))) \quad (58)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x)) \quad (59)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))) \quad (60)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V2z)))))) \quad (61)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q) \vee (p V2r)) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(((\neg(\neg(p V0p))) \Rightarrow (p V0p)))) \quad (74)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0y \in ty\_2Erealax\_2Ereal. \\ & (\forall V1f \in (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V2s \in (2^{A\_27a}). \\ & (\forall V3a \in A\_27a.(((p (ap (c\_2Epred\_set\_2EFINITE \ A\_27a) \ V2s)) \wedge \\ & (p (ap (ap (c\_2Ebool\_2EIN \ A\_27a) \ V3a) \ V2s))) \Rightarrow ((ap (ap (c\_2Eiterate\_2ESum \\ & A\_27a) \ V2s) (\lambda V4x \in A\_27a.(ap (ap (ap (c\_2Ebool\_2ECOND \ ty\_2Erealax\_2Ereal) \\ & (ap (ap (c\_2Emin\_2E\_3D \ A\_27a) \ V4x) \ V3a)) \ V0y) (ap \ V1f \ V4x)))) = (ap \\ & (ap \ c\_2Erealax\_2Ereal\_add (ap (ap (c\_2Eiterate\_2ESum \ A\_27a) \\ & V2s) \ V1f)) (ap (ap \ c\_2Ereal\_2Ereal\_sub \ V0y) (ap \ V1f \ V3a)))))))))) \end{aligned}$$