

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(3)

Definition 13 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 15 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2E$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2F).$

Definition 17 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2)$

Definition 18 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$

Definition 19 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(4)

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal$$
(5)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(6)

Definition 20 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}$$
(7)

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})$$
(8)

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})}$$
(9)

Definition 21 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Erealax_2Ereal)$

Definition 22 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 23 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota.(ap\ (c_2Eiterate_2Eiterate\ A_27a\ ty_2Erealax_2Ereal))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{10}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{11}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{12}$$

Definition 24 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{13}$$

Definition 25 We define $c_2Eiterate_2Emonoidal$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap\ (ap\ c_2Eiterate_2Eiterate\ A_27a\ ty_2Erealax_2Ereal))$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0op \in ((A_27a^{A_27a})^{A_27a}).((p\ (ap\ (c_2Eiterate_2Emonoidal \\
& A_27a)\ V0op)) \Rightarrow (\forall V1f \in (A_27a^{A_27b}).(\forall V2a \in A_27b. \\
& (\forall V3s \in (2^{A_27b}).((ap\ (ap\ (ap\ (c_2Eiterate_2Eiterate\ A_27b \\
& A_27a)\ V0op)\ V3s)\ (\lambda V4x \in A_27b.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& A_27a)\ (ap\ (ap\ (c_2Emin_2E_3D\ A_27b)\ V4x)\ V2a))\ (ap\ V1f\ V4x))\ (ap \\
& (c_2Eiterate_2Eneutral\ A_27a)\ V0op)))))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& A_27a)\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V2a)\ V3s))\ (ap\ V1f\ V2a))\ (ap \\
& (c_2Eiterate_2Eneutral\ A_27a)\ V0op))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c_2Eiterate_2Eneutral\ ty_2Erealax_2Ereal)\ c_2Erealax_2Ereal_add) = \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (c_2Eiterate_2Emonoidal\ ty_2Erealax_2Ereal)\ c_2Erealax_2Ereal_add)) \\
& \tag{21}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in ty_2Erealax_2Ereal. \\
& (\forall V1s \in (2^{A_27a}).(\forall V2a \in A_27a.((ap\ (ap\ (c_2Eiterate_2ESum \\
& A_27a)\ V1s)\ (\lambda V3x \in A_27a.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal) \\
& (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V3x)\ V2a))\ V0b)\ (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal) \\
& (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2a)\ V1s))\ V0b)\ (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))))
\end{aligned}$$