

# thm\_2Eiterate\_2ESUM\_INJECTION (TMLj3cU327EJAduG5Zhpi16vqBKPfDArekF)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_EF$

**Definition 7** We define  $c\_2Ecombin\_2E\_o$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

**Definition 8** We define  $c\_2Ebool\_2E\_EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 9** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (3)$$

**Definition 12** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Emin\_2E40\ x\ s))$ .

**Definition 13** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 14** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E21\ s))$ .

**Definition 15** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p\ (ap\ P\ x))$  then  $(the\ (\lambda x.x \in A \wedge P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 16** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_2Emin\_2E40\ op))$ .

**Definition 17** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V1x \in A\_27a.(ap\ (c\_2Eiterate\_2Eneutral\ op\ x))$ .

**Definition 18** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.c\_2Ebool\_2E21\ t1\ t2)))$ .

**Definition 19** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V1x \in A\_27a.(ap\ (c\_2Eiterate\_2Esupport\ f\ x))$ .

**Definition 20** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V1x \in A\_27a.(ap\ (c\_2Eiterate\_2EITSET\ op\ x))$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (4)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (6)$$

**Definition 21** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ a))$ .

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Erealax\_2Ereal})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (7)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (9)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 24** We define  $c\_2Eiterate\_2ESum$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ ty\_2Erealax$

**Definition 25** We define  $c\_2Eiterate\_2Emonoidal$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (ap\ c\_2$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0op \in ((A\_27b^{A\_27b})^{A\_27b}).((p\ (ap\ (c\_2Eiterate\_2Emonoidal \\ & A\_27b)\ V0op)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}).(\forall V2p \in (A\_27a^{A\_27a}). \\ & (\forall V3s \in (2^{A\_27a}).(((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a) \\ & V3s)) \wedge ((\forall V4x \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V4x) \\ & V3s)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ (ap\ V2p\ V4x))\ V3s)))) \wedge (\forall V5x \in \\ & A\_27a.(\forall V6y \in A\_27a.(((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V5x) \\ & V3s)) \wedge ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V6y)\ V3s)) \wedge ((ap\ V2p\ V5x) = \\ & (ap\ V2p\ V6y)))) \Rightarrow (V5x = V6y)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate \\ & A\_27a\ A\_27b)\ V0op)\ V3s)\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b\ A\_27a) \\ & V1f)\ V2p)) = (ap\ (ap\ (ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ A\_27b)\ V0op) \\ & V3s)\ V1f)))))) \end{aligned} \quad (12)$$

Assume the following.

$$(p\ (ap\ (c\_2Eiterate\_2Emonoidal\ ty\_2Erealax\_2Ereal)\ c\_2Erealax\_2Ereal\_add)) \quad (13)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). \\ & (\forall V1p \in (A\_27a^{A\_27a}).(\forall V2s \in (2^{A\_27a}).(((p\ (ap\ ( \\ & c\_2Epred\_set\_2EFINITE\ A\_27a)\ V2s)) \wedge ((\forall V3x \in A\_27a.(( \\ & p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V2s)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & A\_27a)\ (ap\ V1p\ V3x))\ V2s)))) \wedge (\forall V4x \in A\_27a.(\forall V5y \in \\ & A\_27a.(((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V4x)\ V2s)) \wedge ((p\ (ap\ (ap \\ & (c\_2Ebool\_2EIN\ A\_27a)\ V5y)\ V2s)) \wedge ((ap\ V1p\ V4x) = (ap\ V1p\ V5y)))) \Rightarrow \\ & (V4x = V5y)))))) \Rightarrow ((ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27a)\ V2s)\ (ap\ (ap \\ & (c\_2Ecombin\_2Eo\ A\_27a\ ty\_2Erealax\_2Ereal\ A\_27a)\ V0f)\ V1p)) = ( \\ & ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27a)\ V2s)\ V0f)))))) \end{aligned}$$